# Tone Reproduction based on Singular Value Decomposition for High Dynamic Range Imaging

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*Abstract*— This paper presents a tone reproduction method with the appropriate tone and details of high dynamic range images on the conventional low dynamic range display devices. The proposed algorithm mainly consists of image decomposition using singular value decomposition, a singular value based luminance adjustment, and an image composition process. In the final tone reproduction process, the proposed algorithm combines color and luminance components in order to preserve the color consistency. The experimental results show that the proposed method achieves good subjective quality while enhancing the contrast of the image details.

# Keywords-high dynamic range imaging; tone reproduction; tone mapping; singular value decomposition

#### I. INTRODUCTION

The area of high dynamic range (HDR) imaging is an attractive method with the improved image capturing hardware, and it is an image processing method that produces a large dynamic range. There are many practical applications including scientific, medical visualization [1], satellite imagery [2], and digital camera [3]. In particular, the ability to capture the real world luminance in a scene has become a necessary function of digital camera. However, compared with the  $10^8$ :1 range of real world luminance from bright sunlight to starlight, the HDR image display devices, such as CRT, LCD, and LED have a low dynamic range (LDR) of  $10^2$ :1 cd/m<sup>2</sup> [4].

Although HDR monitors have been existed and researched, they are not only rare and costly, but also difficult to calibrate [5]. Therefore, the tone reproduction (or tone mapping) method is necessary to display on the conventional display devices, for which the luminance range should be transformed to a displayable range that below two orders of magnitude. In tone reproduction, the HDR image contains details in the extremely dim and extremely bright regions; therefore it is difficult to preserve the details while compressing the HDR into the LDR. The discrepancy between the wide range of luminance captured by HDR techniques and the small range of luminance reproduced within displayable ranges from the HDR causes an inaccurate display of the images. Due to this discrepancy, the proper tone reproduction techniques are required. These techniques not only transform HDR images into the displayable range, but also preserve the details, considering the characteristics of the original HDR images.

In the previous works, most tone reproduction techniques have focused on compression and quality evaluation for visualizing HDR images. The tone reproduction can be classified as a global operator [6]-[9] which is the same reproduction function is applied in all regions, or as a local operator [10]-[12] which is the different tone reproduction functions are applied throughout the modeling of spatial adaptations.

This paper is organized as follows. Section II provides a basic concept that is used for the proposed algorithm. In Section III, the proposed method is described. Experimental results prove the performance of the proposed method in Section IV. The paper is concluded with an overall discussion in Section V.

## II. PRELIMINARY KNOWLEDGE

The singular value decomposition (SVD) of a rectangular matrix A has many important properties and useful applications [13]-[16]. Without loss of generality, for every  $m \times n$  ( $m \ge n$ ) matrix A, the SVD can be written as

$$A = USV^{T} = \sum_{k=1}^{n} u_{k} s_{k} v_{k}^{T}$$
(1)

where U=[u<sub>1</sub>, u<sub>2</sub>, ..., u<sub>m</sub>]  $\in \mathbb{R}^{m \times m}$  and V=[v<sub>1</sub>, v<sub>2</sub>, ..., v<sub>n</sub>]  $\in \mathbb{R}^{n \times n}$  are two-column orthogonal matrices, and V<sup>T</sup> denotes the transpose of V. S is a diagonal matrix with elements  $s_i$ , i=1,2,...,n. The diagonal elements can be sorted in a decreasing order, and these are the singular values of matrix B:

$$S = \begin{pmatrix} s_1 & & \\ & \ddots & \\ & & s_n \end{pmatrix} = diag\{s_1, s_2, \cdots, s_n\}.$$
 (2)

The matrix S represents the luminance information of the given image. Note that increasing the magnitudes of the singular values of matrix S will increase the image luminance range, whereas lowering the magnitudes will decrease the image luminance range. Therefore, we can



Figure 1. Overall structure of the proposed algorithm.

use a singular value instead of luminance range. For example, the SVD-based contrast enhancement technique is used while preserving the original image characteristic [13, 14]. In other words, SVD-based method will control the dynamic range maintaining the characteristic of the original image.

#### III. PROPOSED METHOD

The basic principle of the proposed algorithm is to perform a luminance adjustment using SVD. The objective of the proposed SVD-based method is to maintain the important features in the transformation from HDR to LDR. The proposed luminance adjustment method works on the luminance component of the image only which is described in the logarithmic space. From the HDR image, the run-length encoded input values are converted to floating point numbers with linear *RGB* values. The *RGB* values are then converted to a CIE *XYZ* color space [17]. The standard matrix in logarithmic space is obtained by

$$L_{i} = \log(0.2126 \times R + 0.7152 \times G + 0.0722 \times B)$$
(3)

where  $L_i$  is the input luminance value and R, G, and B are the red, green, and blue components.

Firstly, we calculate an anchor image, which offers the dynamic range of displayable devices. The process of displaying an HDR image is composed of quantization and mapping as shown in Fig. 1. Since there are too many discrete values in the high dynamic scene, the tone mapping methods must reduce the number of candidate values by quantization. Quantization is a well-known clustering method [18]. Let x(k) with k = 1, 2, ..., N be the luminance elements of the HDR image. A quantizer is represented by an encoder Q, which maps x(k) to an index  $n \in N$ . One of a small collection of mapping values (candidates)  $C = \{c_n ; n \in N\}$  is used for mapping, where N is set to 256, which is the number of displayable levels in the LDR image. The quantizer is described as

$$Q(x(k)) = n, \quad if \|x(k) - c_n\| \le \|x(k) - c_i\| \quad \forall i.$$
(4)

A pixel in the HDR image is assigned to the available candidate close to the original value. A set of all pixels assigned to the same candidates is defined as a cluster of pixels. In other words, all HDR pixels belong to the same cluster are displayed at the same LDR level. Pixels in the cluster of a larger candidate value are expected be brighter than those of a smaller candidate value. Note that the propose method only works in logarithmic space, treats each pixel individually, and uses the scalar quantization.

The HDR log image,  $L_i$ , is decomposed through the use of (1). Then, the proposed method controls the luminance adjustment by using (2). The adjustable singular value can be seen as a solution of a bi-criteria optimization problem. The object is to find the adjustable singular value  $\tilde{s}$  that is close to the singular value of the quantized image  $s_q$ , while also reducing the size of the  $s_i$ . This adjustable singular value can then be used to obtain the composition using (5) below. This adjustable singular value can be formulated as a weighted sum of the two objectives as

$$\tilde{s} = \arg\min_{s} \left| s - s_i \right| + \lambda \left| s - s_q \right|$$
(5)

where *s*, *s<sub>i</sub>*, and *s<sub>q</sub>* are the maximum singular value, the maximum singular value of the original HDR image, and the maximum singular value of the quantized image, respectively. The parameter  $\lambda$  varies over  $[0, \infty]$ , and the solution of (5) gives the optimal trade-off curve between the two objectives. When the squared sum of the Euclidean norm is used, the following analytical solution of (5) can be obtained:

$$\tilde{s} = \arg\min(s - s_i)^2 + \lambda(s - s_q)^2.$$
(6)

The solution to this minimization problem is obtained by

$$\tilde{s} = (1+\lambda)^{-1}(s_i + \lambda s_a). \tag{7}$$

The adjustable singular value  $\tilde{s}$  is a weighted average of  $s_i$  and  $s_q$ . If  $\lambda$  is zero,  $\tilde{s}$  is the singular value of the input image. As  $\lambda$  approaches infinity, it approaches the singular value of the quantized image. Thus, various levels of luminance can be adjusted by simply changing the parameter  $\lambda$ . Therefore, the displayable luminance adjustment  $L_d$  is composed through the use of the optimal singular value ratio to obtain the following:



Figure 2. The quality comparison with Rosette (720×480): (a) Photoreceptor method, (b) Segmentation-based method, (c) Logarithmic method, (d) Photographic method, and (e) Proposed method. Image courtesy of Paul Debevec.



Figure 3. The quality comparison with Memorial (512×768): (a) Photoreceptor method, (b) Segmentation-based method, (c) Logarithmic method, (d) Photographic method, and (e) Proposed method. Image courtesy of Paul Debevec.

$$L_d = U\tilde{S}V^T$$
, where  $\tilde{S} = \frac{\max(\tilde{s})}{\max(s_i)}S$ . (8)

Conventional methods reproduce images by using the luminance information only. However, the reproduced luminance needs to be combined with the color information to produce good quality tone reproduced images. In the proposed algorithm, the modified color and luminance components are combined to obtain the final tone reproduced values in (9), because the color shifts will be minimized if the ratio among the color channels before and after the compression is maintained. The scale of the color components is determined by  $L_d$ , and the ratio among

the color channels (C = R, G, B) is preserved by the fractions  $C_i/L_i$  as in

$$C_d = \left(\frac{C_i}{L_i}\right)^{\gamma} \times L_d \tag{9}$$

where  $C_d$  represents the final tone reproduced *R*, *G*, and *B* values for the display, and  $C_i$  represents the input HDR data with  $L_i$ . To control the amount of saturation in an image, the fraction is fitted with an exponent,  $\gamma$ , resulting in a per-channel gamma correction, which is given as a user parameter in the range of 0 to 1. Experimentally,  $\gamma$  was set as 0.45.

### IV. EXPERIMENTAL RESULTS

The experimental results are comparable in quality and performance to the manually reproduced images. In this simulation, the parameter  $\lambda$  was set to 99. In Figs. 2 and 3, the proposed algorithm is compared to the conventional tone reproduction operators from a subjective quality point of view. With default parameter settings for each technique, the proposed algorithm is compared using four global operators: the photoreceptor method of Reinhard et al [7], the segmentation-based technique of Yee et al [8], the logarithmic method of Drago et al [9], and the photographic method of Reinhard et al [10], are used. Brief explanations of the methods used to compare against the performance of the proposed algorithm are described as follows. In the global operators, the photoreceptor method first mimics the photoreceptor physiology with some user parameters, but also applies a simple local operator in the spatial domain. Next, the segmentationbased method computes the local adaptation luminance for a global operator that makes use of the image segmentation. The logarithmic method is a simple global tone reproduction algorithm that compresses the luminance range according to the base of the logarithm chosen for each pixel. The photographic method has the global and local operators. In this experiment, the photographic method uses a global operator without a Gaussian filter with a parameter estimation technique.

Figs. 2 and 3 show global tone operators that produce good subjective results, but cannot successfully preserve both tone and local details in an image, particularly in the dim and bright regions such as the cropped regions. In Figs. 2(a) and 3(a), the details are enhanced but the tone is lost to low brightness. In Figs. 2(b) and 3(b), the tone and details are washed out in the whole image. The tone is not natural because the details are over enhanced and the tone is missing, as shown in Figs. 2(c)-(d) and 3(c)-(d). In Figs. 2(e) and 3(e), the proposed technique achieves the natural tone and preserves the details simultaneously.

#### V. CONCLUSION

The most important aspect of reproducing an HDR image as an LDR image is to preserve the tone and details of the HDR image without causing undesirable artifacts, such as halo effects. The proposed algorithm adjusts the luminance values of an image in order to preserve the tone and details based on the SVD-based luminance adjustment. By combining the modified color components with the displayable scale, the color consistency is preserved. The experimental results show that the proposed algorithm achieves good subjective results, especially the balance of the details and subtle tones in both dark and bright regions.

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