Study of Polynomial Methods of Finite Differences for the Wavelength Division Multiplex Mesh Networks With Dedicated Optical Path Protection

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***Abstract*-The dedicated protection method uses the spare capacity as a "dedicated" resource and the backup capacity is allocated for the sole use of the connection or link. This method will be used by polynomial methods of the finite differences. The latter provides two methods, the first is the accurate polynomial method and the second is the approximate polynomial one. These methods are arithmetical, solving successfully the survivability problems and used for the verification of their results. The advantage of the polynomial methods is no use of the large matrix which shows the network active links that is used in the linear method. In this paper, the regression analysis is used to study the absolute error of polynomial finite differences methods (statistical method).**

Keywords: finite differences; accurate polynomial; approximate polynomial; dedicated protection; regression.

I. INTRODUCTION

The WDM (Wavelength Division Multiplex) optical mesh networks are high capacity telecommunication networks based on optical technologies that provide routing, grooming and restoration at the wavelength level as well as wavelength based services.

A fiber cut can have massive implications because network planners use more network elements to increase fiber capacity. For a WDM system with many channels on a single fiber, a fiber cut would result in multiple failures, causing many independent systems to fail. The optical path with dedicated 1+1 protection on optical layer of optical WDM mesh networks can perform protection switching faster and more economically. In the present paper the objective is study the absolute error of the polynomial function methods by statistical method. So, we solve the optical path with dedicated 1+1 protection problem using the polynomial methods of the finite differences calculating the final available capacity by two polynomial finite difference methods, the accurate one and the approximate one. Then the absolute error of two methods is calculating. The procedure is executed for a large number of experiments and the regression analysis is used to study the error. Research has been done in relation to the methods and the problems associated with planning, protection and restoration of optical networks. In [1] the authors present OTN (Optical Transport Network) evolution from an operator's point of view, including the history of the transport network, the role of the OTN, and the motivations and requirements for OTN evolution. For a WDM system with many channels on a single fiber, a fiber cut would results multiple failures, causing many independent systems to fail [2][3][5][6][8][9][10]. There are also several approaches to ensure fiber network survivability [2][3][5][6][9]. Network survivability is defined as the capability of a communication network to resist any link or node interruption or disturbance of service, particularly by warfare, fire, earthquake, harmful radiation or other physical or natural catastrophes.The existing methods in solving these problems use special algorithms. We suggest a proposal that it is an approach based on the finite differences polynomial methods and represents the detail algorithm description and its program. The advantage of this approach is that the polynomial methods of finite differences solve the same telecommunication problem (of this kind) by two different methods simultaneously and they can verify each other in accepted tolerances. These methods also can compare with the linear one [11] and it is another verification way.

The following analysis presents the solution of the problems associated with the survival optical networks on the basis of the finite differences and the following problem is solved. The role of the Difference Calculus is in the study of the Numerical Methods. Computer solves these Numerical Methods. The subject of the Difference Equations [6] is in the treatment of discontinuous processes. The network final available capacity is revealed as a difference equation because the final available capacity of the individual working optical fibers is also a difference equation. The reduction of the available capacity of each working optical fiber is a discontinuous process when connection groups of several sizes pass through it. In [10], the authors begin with an overview of the existing strategies for providing transport network survivability and continue with an analysis of how the architectures for network survivability may evolve to satisfy the requirements of emerging networks. In [11], the author presents the finite differences, their methods and their problems when they are used to solve problems of this kind. In [12], two link disjoint paths, a dedicated working path and a shared protection path are computed , for an incoming light path request based on the current network state but the protection approaches to optimize the resource utilization for a given traffic matrix, do not apply because lightpath requests come and go dynamically.

This paper is broken down in the following sections: Section II shows how the finite differences are used for each optical fiber and illustrates the optical fiber final available capacity; Section III describes the problem, its formulation, its algorithm, an example and the proposals with discussion; Section IV draws conclusion and finally ends with the references.

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II. THE OPTICAL FIBER AND THE FINITE DIFFERENCES

Before studying finite differences and their use in optical WDM mesh networks survivability, it is necessary to provide a short comprehensive presentation of the finite differences computation. Let's assume that $y_1, y_2, \ldots, y_n$ is a sequence of numbers in which the order is determined by the index n. The number n is an integer and the y_n can be regarded as a function of n, an independent variable with function domain the natural numbers and it is discontinuous. Such a sequence shows the available capacity reduction of a telecommunication fibre network link between two nodes when the telecommunication traffic of 1,2, …, n source-destination node pairs pass through. It is assumed that the telecommunication traffic unit is the optical channel that is one wavelength (1λ). The telecommunications traffic includes optical connections with their protections. The total connections of a node pair form its connection group. The first order finite differences represent symbolically the connection group of each node pair that passes through a fiber. This connection group occupies the corresponding number of optical channels and it is the bandwidth that is consumed by connections of a node pair through this fiber. The first order finite differences are used to represent the connection groups in optical channels of the node pairs that pass through an optical fiber .An equation of the first order finite differences gives the available capacity of an optical fiber network link when a connection group passes through it. This available capacity is provided for the connection groups of the other node pairs that their connections will pass through this optical fiber. When the first connection group of Δy_1 connections passes through an optical fiber network link with installed capacity of y_1 optical channels the first order finite difference equation gives the available capacity $y_{2\,(y_1+1)}$ which is written as following

$$y_{1+1} = y_1 - \Delta y_1$$

The sequence $\Delta y_1, \Delta y_2, \Delta y_3, \ldots, \Delta y_n$ represents the connection groups that pass through this optical fiber network link .When Δy_1 subtracted from y_1, creates y_2, when Δy_2 subtracted from y_2, creates $y_3, \ldots$, when Δy_n subtracted from y_n creates y_{n+1} which is the total unused available capacity of this optical fiber. Thus the total unused available capacity of each network optical fiber is calculated after n connections groups pass through it. The total unused available capacity of each network optical fiber is also written as a polynomial function, and there are two polynomial function methods. The assessment of the polynomial function coefficients is done with the values that the polynomial function represents for 1, 2,…, n, n +1. The values of the function y_{n+1} for each n must be integral because each value represents optical channels. There are more details in the [11] but for helping the reader we write them again.

The general form of a polynomial function that gives the available capacity of the optical fiber network link after the serving n connection groups, is as follows

$$y_{n+1} = \sum_{r=0}^{n} \alpha_r * (n+1)^r \qquad (1)$$

The assessment of the polynomial function coefficients is done with the values that the polynomial function represents for 1,2,…, *n*, *n* +1 .The values of the function y_{n+1} for each *n* must be integral because each value represents optical channels.

-If the equation (1) is written analytically as follows

$$y_{0+1} = \alpha_0 * (0+1)^0$$
$$y_{1+1} = \alpha_0 * (1+1)^0 + \alpha_1 * (1+1)^1$$
$$y_{2+1} = \alpha_0 * (2+1)^0 + \alpha_1 * (2+1)^1 + \alpha_2 * (2+1)^2$$
$$y_{3+1} = \alpha_0 * (3+1)^0 + \alpha_1 * (3+1)^1 + \alpha_2 * (3+1)^2 + \alpha_3 * (3+1)^3$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$y_{n+1} = \alpha_0 * (n+1)^0 + \alpha_1 * (n+1)^1 + \alpha_2 * (n+1)^2 + \ldots + \alpha_n * (n+1)^n$$

The value of the function has high accuracy of 15 decimal digits. This method is an *accurate* one.

-If the equation (1) is written analytically as follows

$$y_{0+1} = \alpha_0 * (0+1)^0 + \alpha_1 * (0+1)^1 + \alpha_2 * (0+1)^2 + \ldots + \alpha_n * (0+1)^n$$
$$y_{1+1} = \alpha_0 * (1+1)^0 + \alpha_1 * (1+1)^1 + \alpha_2 * (1+1)^2 + \ldots + \alpha_n * (1+1)^n$$
$$y_{2+1} = \alpha_0 * (2+1)^0 + \alpha_1 * (2+1)^1 + \alpha_2 * (2+1)^2 + \ldots + \alpha_n * (2+1)^n$$
$$y_{3+1} = \alpha_0 * (3+1)^0 + \alpha_1 * (3+1)^1 + \alpha_3 * (3+1)^2 + \ldots + \alpha_n * (3+1)^n$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$Y_{n+1} = \alpha_0 * (n+1)^0 + \alpha_1 * (n+1)^1 + \alpha_2 * (n+1)^2 + \ldots + \alpha_n * (n+1)^n$$

The value of the function has reduced accuracy and depends of the value of *n*. This method is *an approximated* one.

They are systems of n+1equations with n+1unknown coefficients. The values of the coefficients depend of the number of the connection groups and the connections of each connection group. The 1st method is more accurate than 2nd one because only one factor is added to new equation when the polynomial degree increases versus one factor is added for all equations respectively.

TABLE 1.THE SYMBOLS OF THIS PAPER

S/N	Symbol	Comments
1	q	The node number
2	p	The edge number
3	G(V,E)	The network graph
4	V(G)	The network node set
5	E(G)	The network edge set
6	2p	The number of working and backup fiber for 1+1 line protection
7	n	The number of source – destination nodes pairs of the network
8	n(i)	The total number of the connection groups that passes through the fiber (*i*) and means that each fiber has different number of connection groups pass through it
9	$n(i)_w$	The number of the working connection groups that passes through the fiber (*i*) and means that each fiber has different number of connection groups pass through it
10	$n(i)_p$	The number of the protection connection groups that passes through the fiber (*i*) and means that each fiber has different number of connection groups pass through it
11	K	The number of the wavelengths channels on each fiber that is the WDM system capacity
12	Cinst	The total installed capacity
13	Cav	The total available capacity
14	Cw	The total working capacity
15	Cpr	The total protection capacity
16	C_b	The total busy capacity
17	$\Delta y_{w,i,j}$	The first order of finite difference that corresponds to a group of working optical connections that pass through the optical fiber ***i*** with serial number ***j*** respectively.
18	$\Delta y_{pr,i,j}$	The first order of finite difference that corresponds to a group of protection optical connections that pass through the optical fiber ***i*** with serial number ***j*** respectively.

III. THE PROBLEM AND ITS SOLUTION

A. The problem

The network topology and other parameters are known as WDM and optical fiber capacity, one optical fiber per link with an extension to a 1+1 fiber protection system. So this network is characterized by one working fiber per link, edges of two links, links of two optical fibers, one for working and one for protection. The connections are lightpaths originating in the source nodes and terminating at the destination nodes proceeding from preplanned optical working paths. Additionally, the same number of optical paths is preselected for the preplanned fully disjoint backup paths, (1+1 dedicated protection connection). So the connections are protected. The connections of the same node pair form a group along the network. The preplanned protection paths do the dedicated protection of the connection groups. So a suitable number of wavelengths per link along the network uses. The solution is the calculation of the final available capacity of the network for a given table. This table contains the number of the node pairs, the node pairs and the number of the connections of each node pair when their working and protection paths are preplanned. This procedure is executed for polynomial accurate finite difference method and for the corresponded approximate on and the absolute error is calculated. In [11] I have proved that the linear finite difference method (which is an ILP method with the disadvantage the large matrix A (2pxn) which occupies large memory and has difficult treatment)) and the polynomial accurate method are not introducing errors in the solution of the problem. The polynomial approximate one does it. The absolute error is calculated statistically when the procedure is executed for a large number of experiments and the number of the connections of each node pair takes its value by a random number generation with adjusted maximum value. The maximum value must be small in comparison with the WDM and fiber system capacity so that network is a strictly non blocking network, in which it is always possible to connect any node pair, regardless of the state of the network. So all requests for connection are satisfied and form connections. The regression analysis is used to study this absolute error. The regression analysis focuses more on how much each curve of table 2 is better fitting to the error data that introduced by the approximate method. These fitted curves can aid for data visualization, to infer values of a function where no data are available, and to summarize the relationships among the variables. The curves of the regression analysis are in the table 2.

B. The formulation

The network is assumed to be an optical mesh network with the circuit switched (or packet switched but the packets are adjusted to follow preplanned paths) as a graph. Each vertex represents the central telecommunications office (CO) with the OXC while each edge represents two links. Each edge link has a couple of optical fibers. All optical fibers have the same capacity as the WDM system. All nodes are identical. The numbers of working and protection connections that pass through each optical fiber are different. Two finite difference polynomial equations are calculated for each optical fiber, one for polynomial accurate method and one for the polynomial approximate one. For all network links, the general equation of the polynomial function has two column matrices, the left one that is equals with the right one. When all connections have been set up then each element of the column matrix must be greater or equal to zero. In other cases some connections are not possible. The total final available capacity of the network for the polynomial methods is given by the equation (2). This is the formulation of the *polynomial function* method problem.

$$\sum_{i=1}^{2p} y_{i,n(i)+1} = \sum_{i=1}^{2p} \sum_{r=0}^{n(i)} \alpha_{i,r} * (n(i) +1)^r \qquad (2)$$

$$y_{i,n(i)+1} >=0 \quad ,n(i)>0$$

The curves of the regression analysis are in the table 2.

TABLE 2.THE CURVES OF THE REGRESSION ANALYSIS

CURVE NAME	DISTRIBUTION
NORMAL	$Y=A*\exp(((X-B)^2)/C)$
LOG NORMAL	$Y=A*\exp(((\ln(X)-B)^2)/C)$
PARABOLA	$Y=A+B*X+C*X^2$

C. The algorithm

Our algorithm describes the operation of the WDM optical fiber mesh network and we investigate the polynomial finite difference methods. TURBO PASCAL is used to program the model [4]. The algorithm has the following steps and phases.

First step *Network parameters*

Initially the following data are known: network topology, node number, edge number, link number per edge, working optical fiber number per link, protection optical fiber number per link, wavelength number per optical fiber, optical fiber numbering. This information allows the computer to draw a graph and an OXC is on the vertex of the graph [10]. Each edge corresponds to two links with opposite direction to each other. All fibers have the same wavelength number and all links the same fiber number. The computer reads the adjacency matrix and is informed about the network topology.

Second step *Connection selections*

In this step, the number of the connection node pair, the connection node pair selection for connections and the desired connection group size are done. The preplanned working and the protection optical paths for connections of every node pair are also provided. When many experiments are executed a random number generator with adjusted maximum value (max) is activated to give the connection group size.

Failure-free Network Phase

Third step *Wavelength allocation*

In this step, wavelength allocation is initiated. A working connection starts from the source node and progresses through the network occupying a wavelength on each optical fiber and switch to another fiber on the same or other wavelength by OXC, according to its preplanned working optical path up to arrive at the destination node. Simultaneously, the protection connection starts from the source node and progresses through the network occupying a wavelength on each optical fiber and switch to another fiber on the same or other wavelength by OXC, according to its preplanned protection optical path up to arrive at the destination node. So there is full and dedicated protection for this connection. The number of connections of each node pair is equal to its connection group size. After a connection (working as well as protection) has been established, the available capacity is also calculated under both methods of the finite differences. Thus the available capacity of the one method is compared to available capacity of the other method for one connection.

Forth step. *Presentation of the finite differences*

The total available capacity of each optical fiber is calculated and represented under both polynomial methods of finite difference. Thus the total fiber capacity available under the first method is compared to the total fiber capacity available of the other method for all connections.

Fifth step. *Results and comparisons thereof*

Having the desired connection group size the total results are computed under each finite difference method that are the total sum of the individual connection group size, the total installed capacity, the total protection capacity, the total busy capacity and the available residual capacity. These results of the accurate method are compared to the results of the approximated method. The results of these methods must be equal or to have the desired tolerances. In other cases the worst results are rejected.

The *max* value must be small in comparison with the WDM and fiber system capacity so that network is a strictly non blocking network, in which it is always possible to connect any node pair, regardless of the state of the network. So all requests for connection are satisfied and form connections.

Network with failure Phase

When a failure occurs and a link is cut, the optical fibers of this link are also cut and the optical fiber protection 1+1 and the network topology change. The connection groups that passed through the cut link are also cut and the restoration is carried out passing through the preplanned protection paths of other links. The computer is informed of the cut link and modifies suitably the network parameters. The cut optical fiber sets its wavelengths to zero. The connection groups that passing through the cut link set their using wavelengths to zero and through the others to free.

D. Example

The network and the results are presented shortly because the paper must be short. The topology of the network is presented by the graph G (V, E). The vertex set has q=6 elements and the edge set has p=9 elements. Each edge is an optical link of two directions with one working fiber for each direction. Thus there are 2*p=2*9=18 optical fibers. Connection groups transverse the mesh network and correspond to *n* source-destination node pairs. WDM system capacity has 30 wavelengths. The accurate polynomial function method and the approximated polynomial function are presented.

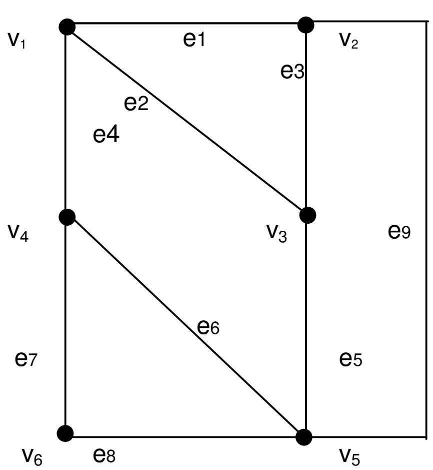

Figure 1.The mesh topology of the network.

TABLE 3.ORDER, SIZE, WORKING PATH, PROTECTION PATH OF EACH NODE PAIR

Node Pair [Si,Di]	Node pair $[v_i, v_j]$	Working Path	Protection Path	Group size
$[S_1, D_1]$	$[v_1, v_2]$	v_1, v_2	v_1, v_3, v_2	1
$[S_2, D_2]$	$[v_1, v_3]$	v_1, v_3	v_1, v_2, v_3	2
$[S_3, D_3]$	$[v_1, v_5]$	v_1, v_3, v_5	v_1, v_2, v_5	5
$[S_4, D_4]$	$[v_2, v_3]$	v_2, v_3	v_2, v_1, v_3	2
$[S_5, D_5]$	$[v_2, v_4]$	v_2, v_1, v_4	v_2, v_5, v_4	2
$[S_6, D_6]$	$[v_2, v_5]$	v_2, v_5	v_2, v_1, v_4, v_5	3
$[S_7, D_7]$	$[v_3, v_4]$	v_3, v_1, v_4	v_3, v_5, v_4	1
$[S_8, D_8]$	$[v_3, v_6]$	v_3, v_5, v_6	v_3, v_1, v_4, v_6	4
$[S_9, D_9]$	$[v_4, v_1]$	v_4, v_1	v_4, v_5, v_3, v_1	1
$[S_{10}, D_{10}]$	$[v_4, v_5]$	v_4, v_5	v_4, v_6, v_5	2
$[S_{11}, D_{11}]$	$[v_5, v_4]$	v_5, v_4	v_5, v_6, v_4	5
$[S_{12}, D_{12}]$	$[v_6, v_1]$	v_6, v_4, v_1	v_6, v_5, v_3, v_1	2

The problem is solved for *n=12 of 30 possible* connection groups. These have their order and sizes for each source-destination node pair, their working paths and their protection paths as shown in *table 3*. The results with finite differences are showed. It is obvious that the dedicated path protection mechanisms use more than 100% redundant capacity because their lengths are longer than their working paths. The total length of working paths is seventeen, (17) and the total length of protection paths is twenty-five, (25).Similarly for the same connections requested group size the capacity that is used by the protection paths is larger than the corresponding working paths.

The synoptic presentation is used for the finite difference tables. So the higher order finite differences and the number of connection groups that pass through each optical fiber are showed in the table 4. (Fiber, i) shows the optical fibers. The n(i) shows the number of the connection groups that pass through each optical fiber. The $\Delta^{m(i)}y_i$ the order finite differences with m(i)=1, 2, 3, 4, 5 of the fiber i. The intermediate order finite differences are not showed. The 16teen has only 1st order differences.

TABLE 4.THE HIGHER ORDER FINITE DIFFERENCES AND THE NUMBER OF CONNECTION GROUPS THAT PASS THROUGH EACH OPTICAL FIBER

Fiber,i	n(i)	m(i)	$\Delta^{m(i)}y_i$
1	3	3	2
2	3	3	1
3	4	4	8
4	4	4	-10
5	2	2	3
6	1	1	1
7	4	4	-8
8	2	2	-1
9	3	3	7
10	2	2	-1
11	3	3	3
12	3	3	5
13	2	2	2
14	2	2	3
15	2	2	-4
16	2	2	0
17	3	3	6
18	0	0	0

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When none group goes through the optical fiber, then the degree of the polynomial function is 0, when one group goes through, then the degree of the polynomial function is 1, when two groups go through, then the degree of the polynomial function is 2, etc.

The polynomials that calculate the available capacity of each optical link for the accurate (first) method for all possible values up to four and for the following cases are represented. The number, the order and the size of $\Delta y_{i,j}$ are critical.

-If no one-connection group passes through an optical link the polynomial function is constant, etc.

$\underline{y_{j,0+1} = \alpha_0 * (0+1)^0}$

$y_{j,0+1} = 30.$

-If only one-connection group passes through an optical link the polynomial function is of the first degree.

$\underline{\Delta y_{i,1}\ ,\quad y_{i,1+1} = \alpha_0 * (1+1)^0 + \alpha_1 * (1+1)^1}$

1 $\quad y_{i,1+1} = 30*(1+1)^0 - (1/2)*(1+1)^1 = 29$

2 $\quad y_{i,1+1} = 30*(1+1)^0 - (2/2)*(1+1)^1 = 28$

3 $\quad y_{i,1+1} = 30*(1+1)^0 - (3/2)*(1+1)^1 = 27$

4 $\quad y_{i,1+1} = 30*(1+1)^0 - (4/2)*(1+1)^1 = 26$

-If only two-connection groups pass through an optical link the polynomial function is of the second degree.

$\underline{\Delta y_{i,1}\ ,\ \Delta y_{i,2}\ ,\ y_{i,1+1} = \alpha_0 * (2+1)^0 + \alpha_1 * (2+1)^1 + \alpha_2 * (2+1)^2}$

1 , 1, $y_{i,2+1} = 30*(2+1)^0 - 0.5*(2+1)^1 - 5.55555555555429E\text{-}2 * (2+1)^2 = 28$

2 , 1, $y_{i,2+1} = 30*(2+1)^0 - 1.0*(2+1)^1 - 0.00000000000000E\text{+}0 * (2+1)^2 = 27$

1 , 2, $y_{i,2+1} = 30*(2+1)^0 - 0.5*(2+1)^1 - 1.66666666667420E\text{-}1 * (2+1)^2 = 27$

3 , 1, $y_{i,2+1} = 30*(2+1)^0 - 1.5*(2+1)^1 + 5.55555555555429E\text{-}2 * (2+1)^2 = 26$

2 , 2, $y_{i,2+1} = 30*(2+1)^0 - 1.0*(2+1)^1 - 1.11111111111086E\text{-}1 * (2+1)^2 = 26$

1 , 3, $y_{i,2+1} = 30*(2+1)^0 - 0.5*(2+1)^1 - 2.77777777777828E\text{-}1 * (2+1)^2 = 26$

4 , 1, $y_{i,2+1} = 30*(2+1)^0 - 2.0*(2+1)^1 + 1.11111111111086E\text{-}1 * (2+1)^2 = 25$

3 , 2, $y_{i,2+1} = 30*(2+1)^0 - 1.5*(2+1)^1 - 5.55555555555429E\text{-}2 * (2+1)^2 = 25$

2 , 3, $y_{i,2+1} = 30*(2+1)^0 - 1.0*(2+1)^1 - 2.22222222221720E\text{-}1 * (2+1)^2 = 25$

1 , 4, $y_{i,2+1} = 30*(2+1)^0 - 0.5*(2+1)^1 - 3.88888888886870E\text{-}1 * (2+1)^2 = 25$

4 , 2, $y_{i,2+1} = 30*(2+1)^0 - 2.0*(2+1)^1 + 0.00000000000000E\text{+}0 * (2+1)^2 = 24$

3 , 3, $y_{i,2+1} = 30*(2+1)^0 - 1.5*(2+1)^1 - 1.66666666667420E\text{-}1 * (2+1)^2 = 24$

2 , 4, $y_{i,2+1} = 30*(2+1)^0 - 1.0*(2+1)^1 - 3.33333333333485E\text{-}1 * (2+1)^2 = 24$

e.t.c

The following matrix provides the residual capacity of all optical fibers, its dimension is (18x1) and the first method or the accurate one is written for all network links as follows.

$y_{1,3+1}$	30 – 0.5*4 -0.167*16-0.05208*64		22
$y_{2,3+1}$	30 -1.00 *4 - 0.111*16-0.0191*64		23
$y_{3,4+1}$	30 - 0.5 *5 - 0.167*25-0.05208*125 +0.00508*625		20
$y_{4,4+1}$	30 – 0.5*5-0.3888*25+0.03472*125-0.0001888*625		22
$y_{5,2+1}$	30 - 1 *3 - 0.11111*9		26
$y_{6,1+1}$	30 - 0.5*2		29
$y_{7,4+1}$	30 - 1*5 - 0.22222*25+0.0243*125-0.003972*625		20
$y_{8,2+1}$	30 - 0.5*3 -0.167*9		27
$y_{9,,3+1}$ =	30 - 2.5 *4 + 0.1667*16-0.041667*64	=	20
$y_{10,2+1}$	30 - 0.5 *3 - 0.167*9		27
$y_{11,3+1}$	30 - 1.5 *4 + 0.055*16-0.013889*64		24
$y_{12,3+1}$	30 - 1*4+0*16-0.0625*64		22
$y_{13,2+1}$	30 – 2*3 -0*9		24
$y_{14,2+1}$	30 - 2.5*3 + 0.0555*9		23
$y_{15,2+1}$	30 - 2*3 - 0.333*9		21
$y_{16,2+1}$	30 - 1*3 - 0.1111*9		26
$y_{17,3+1}$	30 – 2.5*4 +0.05555*16-0.013889*64		20
$y_{18,0+1}$	30		30

This method is an accurate one but if all coefficients are not written completely with 15 digits there are errors in the results. The above ones are rounded to the closest integer to agree with the real ones.

The polynomials that calculate the available capacity of each optical link for the approximated (second) method for all possible values up to four and for the following cases are represented. The number, the order and the size of $\underline{\Delta y_{i,j}}$ are critical.

-If no one-connection group passes through an optical link the polynomial function is constant.

$\underline{y_{j,0+1} = \alpha_0 * (0+1)^0}$

$y_{j,0+1} = 30.$

-If only one-connection group passes through an optical link the polynomial function is of the first degree.

$\underline{\Delta y_{i,1}\ ,\quad y_{i,1+1} = \alpha_0 * (1+1)^0 + \alpha_1 * (1+1)^1}$

1 $\quad y_{i,1+1} = (30+1)*(1+1)^0 - 1*(1+1)^1 = 29$

2 $\quad y_{i,1+1} = (30+2)*(1+1)^0 - 2*(1+1)^1 = 28$

3 $\quad y_{i,1+1} = (30+3)*(1+1)^0 - 3*(1+1)^1 = 27$

4 $\quad y_{i,1+1} = (30+4)*(1+1)^0 - 4*(1+1)^1 = 26$

-If only two-connection groups passes through an optical link the polynomial function is of the second degree.

$\underline{\Delta y_{i,1}\ ,\ \Delta y_{i,2}\ ,\ y_{i,1+1} = \alpha_0 * (2+1)^0 + \alpha_1 * (2+1)^1 + \alpha_2 * (2+1)^2}$

1 , 1, $y_{i,2+1} = 31*(2+1)^0 - 1.0*(2+1)^1 + 0.000*(2+1)^2 = 28$

2 , 1, $y_{i,2+1} = 33*(2+1)^0 - 3.5*(2+1)^1 + 0.500*(2+1)^2 = 27$

1 , 2, $y_{i,2+1} = 30*(2+1)^0 + 0.5*(2+1)^1 - 0.500*(2+1)^2 = 27$

3 , 1, $y_{i,2+1} = 35*(2+1)^0 - 6.0*(2+1)^1 + 1.000*(2+1)^2 = 26$

2 , 2, $y_{i,2+1} = 32*(2+1)^0 - 2.0*(2+1)^1 - 0.000*(2+1)^2 = 26$

1 , 3, $y_{i,2+1} = 29*(2+1)^0 + 2.0*(2+1)^1 - 1.000*(2+1)^2 = 26$

4 , 1, $y_{i,2+1} = 37*(2+1)^0 - 8.5*(2+1)^1 + 1.500*(2+1)^2 = 25$

3 , 2, $y_{i,2+1} = 34*(2+1)^0 - 4.5*(2+1)^1 + 0.500*(2+1)^2 = 25$

2 , 3, $y_{i,2+1} = 31*(2+1)^0 - 0.5*(2+1)^1 - 0.500*(2+1)^2 = 25$

1 , 4, $y_{i,2+1} = 28*(2+1)^0 + 3.5*(2+1)^1 - 1.500*(2+1)^2 = 25$

4 , 2, $y_{i,2+1} = 36*(2+1)^0 - 7.0*(2+1)^1 + 1.000*(2+1)^2 = 24$

3 , 3, $y_{i,2+1} = 33*(2+1)^0 - 3.0*(2+1)^1 + 0.000*(2+1)^2 = 24$

2 , 4, $y_{i,2+1} = 30*(2+1)^0 + 1.0*(2+1)^1 - 1.000*(2+1)^2 = 24$

e.t.c

The following matrix provides the residual capacity of all optical fibers, its dimension is (18x1) and the second method or the approximated one is written for all network links as follows.

$y_{1,3+1}$	32 – 3.1574*4 +1.5*16-0.3336*64		22.02
$y_{2,3+1}$	33 - 3.8241*4 + 1*16-0.1669*64		23.022
$y_{3,4+1}$	40 - 19.8339 *5 + 13.1667*25-3.6661*125 +0.3333*625		20.048
$y_{4,4+1}$	12 + 35.33*5-22.083*25+5.1671*125-0.4167*625		22.032
$y_{5,2+1}$	32 - 2*3 +0*9		26
$y_{6,1+1}$	31 - 1*2		29
$y_{7,4+1}$	20 + 21.662 *5 -15.1666*25+3.8338*125-0.3334*625		20.016
$y_{8,2+1}$	30 + 0.5*3 -0.5*9		27
$y_{9,3+1}$	= 46 – 23.824*4 +9*16-1.167*64	=	20.016
$y_{10,2+1}$	30 + 0.5*3 - 0.5* 9		27
$y_{11,3+1}$	38 - 11.494*4 + 4*16-0.5*64		24.024
$y_{12,3+1}$	38 - 12.6574*4+5.5*16-0.8336*64		22.02
$y_{13,2+1}$	36 – 7*3 +1*9		24
$y_{14,2+1}$	38 - 9.5*3+1.5*9		23
$y_{15,2+1}$	33 - 2.5*3 - 0.5*9		21
$y_{16,2+1}$	32 - 2*3 + 0*9		26
$y_{17,3+1}$	42 – 16.824*4+5.5*16-0.667*64		20.016
$y_{18,0+1}$	30		30

The absolute error for all fibers and for both methods is the following.

Error abs $y_{1,4}$		0.02
Error abs $y_{2,4}$		0.022
Error abs $y_{3,5}$		0.048
Error abs $y_{4,5}$		0.032
Error abs $y_{5,3}$		0
Error abs $y_{6,2}$		0
Error abs $y_{7,5}$		0.016
Error abs $y_{8,3}$		0
Error abs $y_{9,,4}$	=	0.016
Error abs $y_{10,3}$		0
Error abs $y_{11,4}$		0.024
Error abs $y_{12,4}$		0.02
Error abs $y_{13,3}$		0
Error abs $y_{14,3}$		0
Error abs $y_{15,3}$		0
Error abs $y_{16,3}$		0
Error abs $y_{17,4}$		0.016
Error abs $y_{18,1}$		0

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If the degree of polynomials increases then the above writing of the polynomial numerical coefficients has error because they are difficult to be represented.

The total available capacity for the accurate (first) method is $Cavp_1$=426 wavelengths and for the approximated (second) method is $Cavp_2$=426.214 wavelengths. Thus, the total busy capacity for the accurate (first) method is given C_{b1} =114 wavelengths and for the approximated (second) method is given C_{b2} =113.786 wavelengths. The network installed capacity is Cinst=18*30=540 wavelengths. So the following sum is valid Cb1+Cav=Cinst or 114+426=540. The absolute error between the accurate and approximated methods is $\Delta C=|(Cav_2 - Cav_1)|=0.214$.

For the best study of the precision, 100000 experiments are executed. The average is 0.324037, the variance is 0.005072. The width of the variance is 0.47999999515712 with the minimum error equal to 0.088000000454485 and maximum error 0.56799999997019. So the range of the errors is separated in nine intervals (classes) with width 0.06. The classes are showed in the table 5 second column (class). The median of each class is showed in the table 5, third column (x_i). For the better representation of the data that are measured, the graphical method is used figure 2, curve absolute (the measured distribution) and figure 3, curve athristikh (the measured accumulated distribution). The parameters of the measured frequencies are showed in the table 6, second column (absolute).The parameters ASYM and CURV mean the coefficient of asymmetry and the coefficient of curve respectively. After it is tried to understand, model and analyze the relationship between a dependent variable (error frequency) and one independent variable (error class that is showed by its median). It is a regression analysis. The curves of figure 2 are the normal distribution and parabola one. The curves of figure 3 are the log normal distribution and the parabola one. The parameters of the curves of figure 2 are showed in the table 6. The coefficient of correlation between the random variable error class and error frequency is a quantitative index of association between these two variables. In its squared form, as a coefficient of determination R^2 indicates the amount of variance in the criterion variable error frequency that is accounted by the variation in the predictor variable error class. These coefficients for the curves of the Figures 2 and 3 showed in the next Table 7.

TABLE 5.THE CLASSES, THE MEDIAN OF EACH CLASS, THE RELATIVE AND ACCUMULATED RELATIVE FREQUENCIES OF THE ERRORS

S/N	CLASS	x_i	fi'	Fi'
1	(0.054,0.114]	0.084	0.026	0.026
2	(0.114,0.174]	0.144	1.28	1.306
3	(0.174,0.234]	0.204	8.568	9.874
4	(0.234,0.294]	0.264	24.003	33.877
5	(0.294,0.354]	0.324	32.386	66.263
6	(0.354,0.414]	0.384	23.652	89.915
7	(0.414,0.474]	0.444	8.795	98.71
8	(0.474,0.534]	0.504	1.257	99.967
9	(0.534,0.594])	0.564	0.033	100

TABLE 6.THE PARAMETERS OF THE ABSOLUTE, NORMAL AND PARABOLA DISTRIBUTIONS

PARAMETER	ABSOLUT	NORMAL	PARABOL
AVERAGE	0.324037	0.3251	0.324037
DEVIATION	0.005072	0.0041	0.005072
ST.DEVIATION	0.071217	0.064032	0.071217
ASYM	0.0061	0	-0.20141
CURV	2.75117	3	-17.9911

TABLE 7.THE COEFFICIENTS R^2 FOR THE CURVES OF THE FIGURES 2 AND 3

Figure	NORMAL	PARABOLA
2	0.9884	0.7313
Figure	**LOG NORMAL**	**PARABOLA**
3	0.9986	0.9323

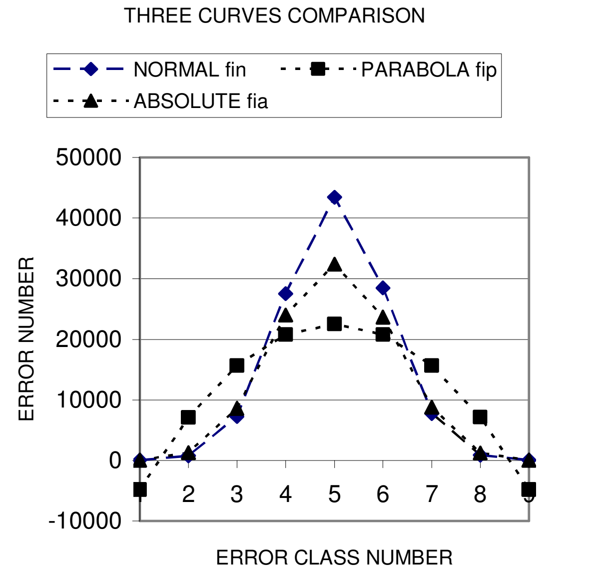

Figure 2.The three distributions.

The curve-fitting techniques will (in most practical cases) produce functions that will not precisely fit all the data points (sometimes none of the points exactly lie on the curve such that the residuals Rin=fin-fia and Rip=fip-fia are existed). These are showed for the curves of figure 2 in the figure 4. There are two fittings with their error patterns. The different two colors are indicating two different fits. It is obvious that the residuals are scattered between negative and positive. This bar chart gives limited visual impressions about the two fitting residuals. We can recognize a random pattern showing us the two fits of the residual distribution for the error class number from 1 to 9. Every abscissa should have two different color bars. The first fit is black and the second white with shadow. We can see that some bars are missing which indicates a zero (or a very small) residual. For example, we may say that at the error class number 1 and 9, the first fit is very accurate (we cannot see the bright black rectangle), while in error class number 5 the black rectangle is larger which means that the first fit function, represented by black color, has a very bad fit there. At the figure 5, the absolute residuals are showed and

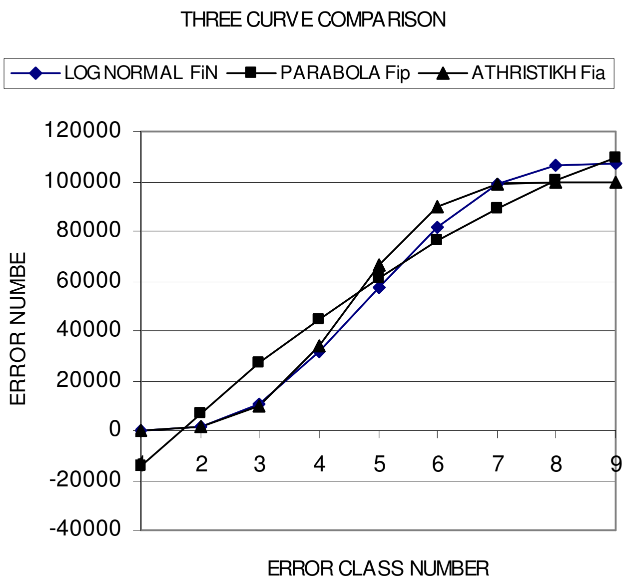

Figure 3.The three distributions (ATHRISTIKH means accumulated in Greek Language).

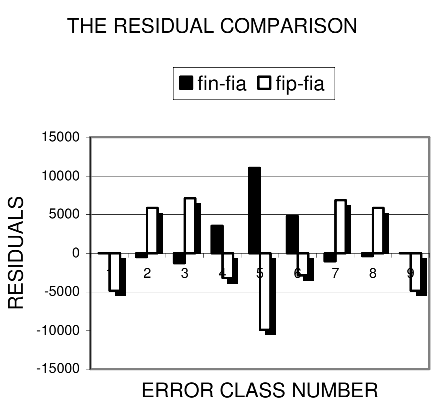

Figure 4.Residuals for curves of figure 2.

the comparison is presented better. At the figure 6, for the better presentation of the fitting, the error class number is replaced by the error grade and the absolute residuals are sorted. The first error grade (1) represents the minimum absolute residuals (or deviations) for everything of the two fits and last one (9) represents the maximum absolute residuals. The error grade number two (2) also represents residuals that come in the second grade for everyone of the two fits, and so on. Therefore, instead of comparing only maximum errors (mostly occurring at different two points) as indications of how good is the fit, we may compare all the 9 grades of errors and find out which fit would sum up the least errors and that would be our required fit. The same analysis could be represented for the curves of the figure 3.

The complexity of this algorithm for the node number *q* depends on the square of the node number and the total number of the requests for connection (s) so it is written as

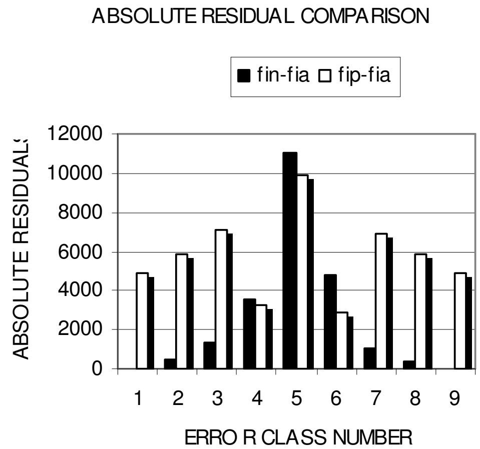

Figure 5.The absolute residuals for the curves of figure 2.

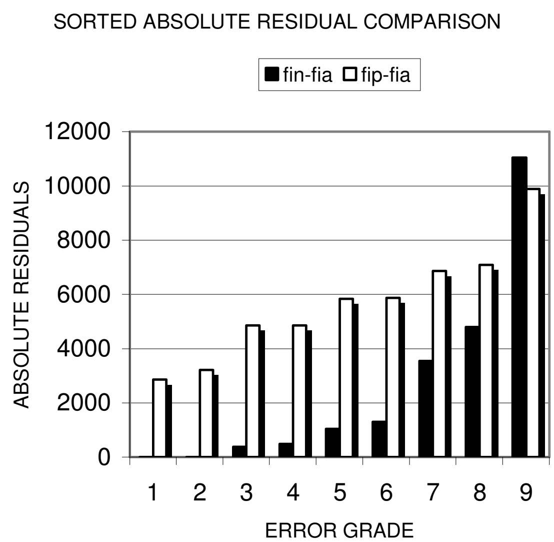

Figure 6.The sorted absolute residuals for the curves of figure 2.

O $(s*q^2)$. Time complexity of that algorithm is 'order q^2, $O(s*q^2)$. Consuming time for 100000 experiments for the accurate method is 16' 43'' 10 and for the approximate one 16' 46'' 46. On a 133MHz computer the approximate method consumes more time than the accurate one to solve the same problem in the same network. The worst consuming time depends of network size for the same computer. The matrix of coefficients of the accurate method is low triangular and of the approximate one is square. So the approximate method needs more memory for solving.

E. Discussion and Proposals

In this protection scheme, when a single failure of a cut link occurs and the main connection also cuts then the connection is routed by backup path.

The use of the polynomial methods of finite differences is possible for the study of the problems related to the protection and restoration of connections and has the advantage versus the linear method that is not using the large matrix with the active links of the network. The active links are the links that pass through the optical paths. For better presentation of this research a short example is used that depicts the results in these methods. The algorithm provides for each source-destination node pair and a desired connection group size, a value of the total available capacity of the network. The connection length depends on the number of hops. The network has a complete protection for optical path connection. It is a switch circuit network (or a packet switch network but the packets are adjusted to pass through preplanned paths) so that one lightpath corresponds to one optical connection. Different wavelengths may be used for each connection in each hop, so that wavelength conversion is used at each node.

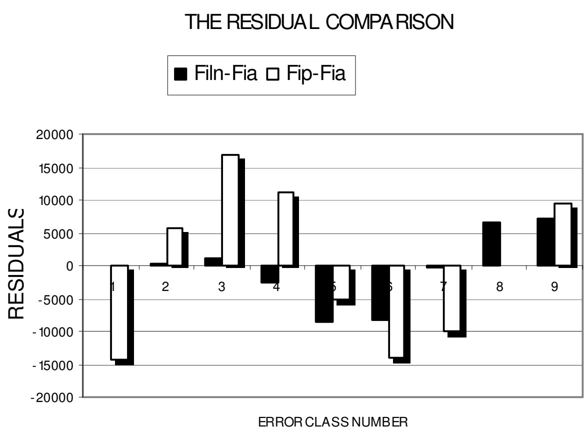

Figure 7.The residuals for the curves of figure 3.

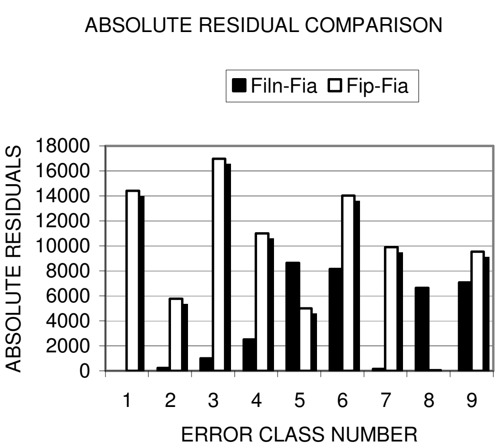

Figure 8.The absolute residuals for the curves of the figure 3.

These methods solve problems with small networks because when the connections groups that pass through a link increases, then the number of the (n+1) equations with n+1 unknown of the system also increases and it is difficult to be solved to calculate the coefficients. The accurate method is easier and more accurate than the approximate one because only one factor is added to each new equation when the polynomial degree increases versus one factor is added for all equations. In Turbo Pascal for the PC, the best precision is 15 decimal digits and some differences that appeared are ought to the difficulty to write all decimal digits for each polynomial coefficient of the polynomial methods. For the precision, the *double* type is used that is a floating point format and provides good dynamic range in addition to high precision.

The coefficient of determination R^2 value is an indicator of how well your data fits a line or curve. This coefficient value is in the range (0, 1). When a distribution (curve) has larger R^2 than another, it has also best dependent for its variables than another one. In other words, if the R^2 value is closer to 1, means more likely your data points are solutions to the equation that defines your curve. This indicates how well your data fits the model you are testing. The normal distribution has best fitting than the parabolic one to the measured distribution and the log normal distribution has also best fitting than the parabolic one to the measured accumulated distribution. These are showed in the figures, 2 and 3. For better presentation of the curve fitting, the residuals are showed in the figures, 4 up to 9.

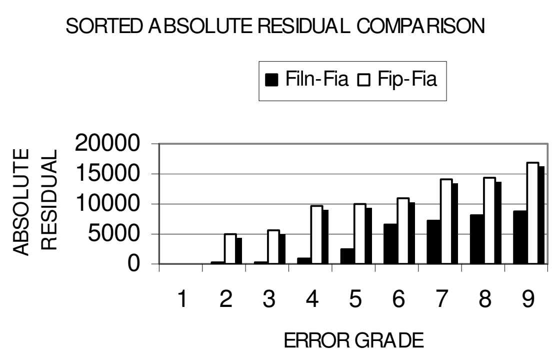

Figure 9.The sorted absolute residuals for the curves of figure 3.

The solution of the optimization problem that means the solution of a problem that more approaches to our demands is more complex. The general solution of this problem is difficult so it is transformed as a NP-complete problem. It is also noted that ILP formulations are practical only for small networks because the number of the equations and the variables should be as less as possible. The statistical method solves the problem but in a long time. For larger networks only heuristic methods are used because they are faster and give a very good, if not optimal, result. A heuristic method tries to solve the optimization problem in one or two steps ignoring whether the solution can be proven to be correct, a good or better solution is produced. So the computation performance is improved obtaining in a short time the solution of this problem at the cost of the accuracy. The simplex method is impractical for larger networks because there are a lot of scenarios, it is an ILP problem and the number of the variables and the number of equations also increase rapidly.

IV. CONCLUSION

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In this paper, the dedicated protection optical path method has been researched on the basis of the polynomial methods of finite differences statistically. The methods of finite differences make it possible to research and study the dedicated protection problems of the optical paths. They provide suitable, accurate arithmetical methods to solve telecommunication problems such as planning of a completely protected network, complete protection for any failure occurs on optical path, node or link e. t. c.

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