Performance of Relay-Aided Distributed Beamforming Techniques in Presence of Limited Feedback Information

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***Abstract*—This paper studies the impact of channel signaling resolution on the performance of a (coherent) *distributed beamforming* (DBF) algorithm. This analysis is done in the context of a wireless access network, whose ultimate goal is to give adequate broadband coverage for users inside buildings. In this situation, instead of trying to reach the serving *base station* (BS) directly, we assume that each indoor subscriber receives assistance from a cooperative network that is deployed in its premises. This surrounding cooperative network is formed by a (relative) large number of low-cost *relay nodes* (RNs) with only one antenna. To simplify the analysis, communication in the first link (i.e., from the subscriber's terminal to RNs) is assumed costless, making the bottleneck lay in the second link (i.e., from RNs to serving BS). To carry out the analysis, a suitable closed-form approximation for the outage probability that correspond to a given received *signal-to-noise* power *ratio* (SNR) threshold is derived. Our analysis reveals that the power gain sacrificed when using a small amount of phase feedback information is not considerable in the light of the performance loss that is observed.**

***Keywords*-Cooperative Communications; Distributed Beamforming; Limited Feedback; Outage Probability; Relay Nodes.**

I. Introduction

Standardization activities on wireless communication networks advance rapidly, fueling the creation of new research topics to cope with the ever-increasing demand of higher data rates for mobile radio access. Given that most part of current voice calls and data usage takes place inside buildings [1], the provision of adequate broadband coverage in indoor environments is a crucial issue [2], [3]. Since indoor users are often behind walls with high attenuation, the penetration losses that radio signals experience put the mobile terminals in a very disadvantageous position, increasing the energy consumption required for signal transmission and reducing the amount of information that can be transfered effectively. Trying to give a solution to this problem, recent developments showed that such difficulties can be overcome by deploying femtocells (i.e., small and low-power wireless access points that connect mobile devices to the cellular network via the wired connection of the subscriber) [4]. Nevertheless, a rather different approach is analyzed in this work, where we propose to boost the communication performance of indoor users deploying a cooperative network in the premises of the subscriber. This network will be formed by a large number of low-cost *relay nodes* (RNs), that will assist the communication between the mobile terminal (i.e., main transmitter) and the network base station (i.e., main receiver) implementing a *distributed beamforming* (DBF) strategy.

The main idea behind DBF is simple: distribute a common message within many low-power single-antenna RNs, and then coordinate the re-transmission of this information in the direction of the intended destination (configuring a virtual transmit antenna array) [5], [6]. When the main transmitter and the RNs share the same environment (e.g., the same room), the communication in the first link can be carried out with almost no cost (in terms of time and power). In this situation, the bottleneck of this system lies in the second link, and can be mitigated by adjusting the channel response (i.e., amplitudes and phases) that the main receiver sees from each individual RN. This enables the coherent combination of the multiple replicas of the original message at the intended destination. The potential benefit of deploying a DBF scheme is well known in the literature: full diversity benefit and M-fold power gain for M active RNs in the network[1] [7]. However, in absence of *channel state information* (CSI) at the RNs, the use of distributed space-time coding was suggested to obtain full diversity gains in the second link (no power gain is possible in this situation) [8].

The main challenges in a DBF scheme are in the synchronization of the RF carriers of all RNs, and in the estimation of each individual channel gain that the main receiver observes in the second link. An adaptive 1-bit feedback DBF algorithm that tries to solve these problems was developed by Mudumbai *et al.* in [9]. The basic idea behind Mudumbai's DBF algorithm is interesting: make independent random phase adjustment at the RNs in each iteration, and retain only those phases that increase the received *signal-to-noise* power *ratio* (SNR) at the main receiver. Even though Mudumbai's DBF algorithm has shown many interesting convergence properties, in this paper we focus in a rather different approach. We assume that the main receiver has the capability to estimate the individual channels from each RN in the second link (using a N-bits uniform quantizer). Since the locations of RNs remain fixed during the whole data communication, only the phase portions of the channel gains are assumed to take random (unknown) values at the beginning of the phase configuration process. A closed-form approximation for the outage probability of this *deterministic* DBF algorithm is derived. Based on this analysis

[1]The power gain increases to a factor of M^2 if each RN transmits always at full power, independently of the number of active RNs in the network.

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it is possible to conclude that a relatively small amount of phase signaling information (i.e., $N = 3$ phase feedback bits per channel) is sufficient to obtain a performance close to the one observed in presence of perfect channel phase information at the RNs.

The rest of the paper is organized as follows. Section II presents the system model, the assumptions on the limited feedback DBF algorithm, and the details of the performance criterion that will be used to carry out the analysis. Section III provides expression for the distribution of the received SNR, while Section IV presents the numerical results and studies the impact of the number of feedback bits per channel (i.e., N) and the number of active RNs (i.e., M) to the outage probability of the system. Finally, Section V presents the conclusions of the work.

II. System Model

The general layout of our cooperative relaying system is illustrated in Fig. 1. The system consists of a main transmitter, a main receiver, and M active RNs that share the same physical space with the main transmitter (e.g., the same room or office). All devices are equipped with a single transmit/receive antenna (in accordance with the low-cost requirement for RNs). In our system model, main transmitter and RNs operate in a half-duplex mode in a *decode-and-forward* (DF) fashion. Thus, during the first hop of duration T_1, message intended for the main receiver is sent from main transmitter to the nearby RNs. During the second hop of duration T_2, the message is sent from the RNs to main receiver. Attenuation on this first link is assumed to be small, and channel is either static or slowly varying (e.g., line of sight channel model). This makes possible to assume that communication on the first link can be accomplished with (almost) no cost in terms of power and/or time. The long distance between clustered RNs and main receiver implies a large attenuation on the second link, when compared to the attenuation on first link. This situation makes the second hop the bottleneck of the system, and its analysis the main goal of this paper.

As depicted in Fig. 1, a low-rate, reliable, and delay-free feedback channel exists between the main receiver and the active RNs. Main receiver uses this channel to convey a quantized version of the phase adjustment that each RN should apply in transmission (to maximize SNR in reception). In other words, the limited feedback information that main receiver reports is used to establish a *virtual antenna array* (VAA) in the second link. Note that since all RNs share the same physical location with the main transmitter, no multi-hop strategy is able to provide a better performance than the one obtained with a direct connection between main transmitter and main receiver. Therefore, the only valid option to reach the main receiver is that multiple active RNs transmit cooperatively at the same time, focusing the resulting VAA beam toward the direction of the intended destination over the second link.

Based on the above model, the received signal at transmission time interval i is of the form

$$r[i] = H[i]s[i] + n[i], \tag{1}$$

where $H[i]$ is the resulting sum channel, $s[i]$ is the complex modulation symbol, and $n[i]$ refers to an *additive white Gaussian noise* (AWGN) sample. Power control is not applied in the RNs and thus, the total transmit power P_t in the second hop remains fixed during the whole communication.

In case of unitary noise power, the received SNR in the second hop is given by

$$\Gamma[i] = \Big| \sum_{m=1}^{M} \sqrt{\gamma_m[i]}\, w_m[i] e^{j\psi_m[i]} \Big|^2, \tag{2}$$

where $\gamma_m[i]$ represents the received SNR from the m-th RN, $\psi_m[i]$ is the corresponding channel phase response, and $w_m[i]$ is the transmit weight that the m-th RN applies.

In our system model it is also assumed that:

- All devices admit fixed location. Thus, channel is not changing in time and we have $\gamma_m[i] = \gamma_m$, $\psi_m[i] = \psi_m$.
- Weights $w[i]$ are used to make phase adjustments in RNs. Required feedback message for adjustments may be spread over a time span, denoted by $i = 1, 2, \ldots, \mathcal{I}$.
- Performance analysis considers the resulting sum channel when all phase adjustments are done (i.e. after $\mathcal{I}$ time intervals). The corresponding weights in this situation are denoted by w_m (i.e., time index can be dropped since phase adjustments have been done and channel is static).
- Phases ψ_m are not calibrated in RNs, but they behave as independent, random samples. In the analysis we study the performance over any initial phase configuration. Therefore, we assume that phases ψ_m are *independent and identically distributed* (i.i.d.) uniform *random variables* (RVs) that take values on interval $(-\pi, \pi)$.
- To fulfill the accurate timing requirement, RNs monitor the standard synchronization signals either from destination (i.e., main receiver) or source (i.e., main transmitter).

The signal phase shifts that RNs apply easily create frequency selectivity, which is seen as an increased multipath effect in the resulting wireless channel. Yet, in our system model we assume that the synchronization error of RNs is small when compared to symbol length. Then, it is possible to assume that the duration of the effective channel impulse response is not (considerably) increased in this situation.

A. Assumptions on Limited Feedback Scheme

As shown in Fig. 1, in the second stage of the communication there are M active RNs transmitting a common symbol s to the main receiver. In order to maximize the SNR in the main receiver, each RN adjusts its transmission signal using a complex, individual beamforming weight

$$w_m = \sqrt{\frac{P_t}{M}}\, e^{-j\phi_m}, \quad \phi_m \in \mathcal{Q}, \tag{3}$$

$$\mathcal{Q} = \left\{ \frac{2\pi(n-1)}{2^N} : \ n = 1, \ldots, 2^N \right\}. \tag{4}$$

We note that the individual power (i.e., the module of w_m) is selected based on the number of active RNs in the cooperative system, so that total transmission power is always normalized

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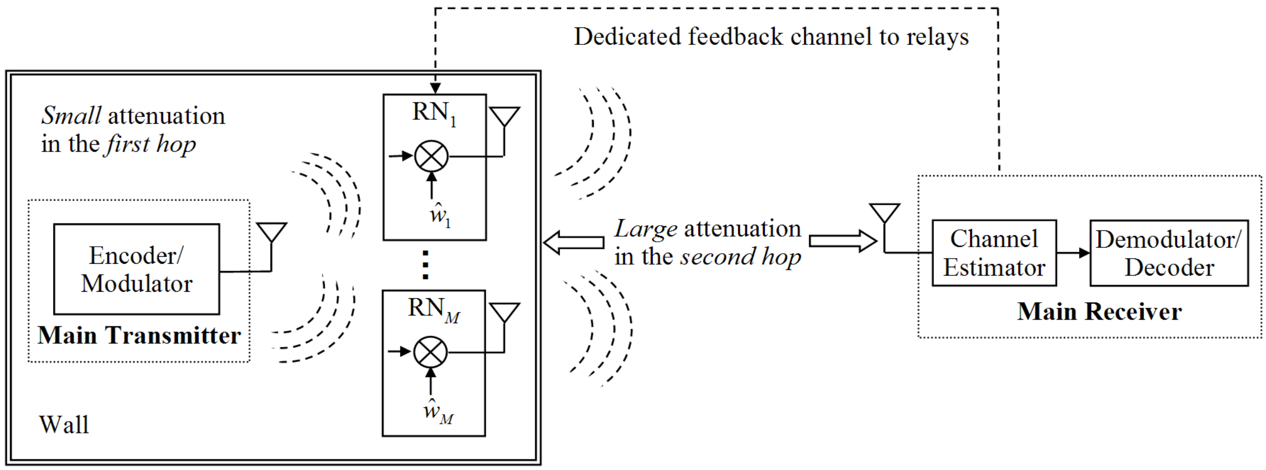

Figure 1. Cooperative relaying system model.

(and fair comparisons are possible). The error free feedback indicating the best index n is provided through the dedicated feedback channel. Number of feedback bits per RN is N.

Phases ϕ_m are selected in the main receiver as follows: Receiver first estimates the phases ψ_m from RN specific reference signals. After that, it selects the phases ϕ_m from quantization set $\mathcal{Q}$ such that $|\theta_m| = |\phi_m - \psi_m|$ is minimized. As a result, adjusted phases θ_m will be uniformly i.i.d. on the interval $(-\frac{\pi}{2^N}, \frac{\pi}{2^N})$ [10]. Thus, phase adjustments are done independently, using a common phase reference at the receiver side.

B. Performance Criterion: Outage Probability

There are two main performance measures that have been defined in the literature to carry out (theoretical) performance analyses: the ergodic capacity and the outage capacity [11]. The ergordic capacity is the long-term average transmission rate, and can be achieved implementing coding schemes that span code words over several coherence time intervals of the fading channel. This measure is feasible for applications with no strict delay constraints. However, in case of constant-rate delay-limited transmissions with coding over a single channel realization, the outage capacity becomes a more appropriate performance indicator. The outage capacity defines the maximum constant rate that can be maintained for a given outage probability.

In practical system implementations, however, a slightly different criterion is widely used. When mobile system performance is evaluated, it is assumed that reception is successful if SNR at the receiver (for the given transmission time interval) is large enough. In other words, a user is said to be supported if its instantaneous received SNR satisfies

$$\Gamma[i] \geq \gamma_0, \tag{5}$$

where the threshold γ_0 is defined to guarantee a certain level of service (for the given transmission rate). In this situation, the statistical performance requirement

$$\Pr\{\Gamma[i] \leq \gamma_0\} = \Pr_{\text{out}}(\gamma_0) \tag{6}$$

is defined as the outage probability for a given target SNR threshold. This is the performance measure that will be used throughout this work.

III. Performance of Distributed Beamforming with Limited Feedback

According to the system model presented in Section II, the relevant expression for SNR is of the form

$$\Gamma[i] = |H[i]|^2 = \frac{P_t}{M}\left|\sum_{m=1}^{M}\sqrt{\gamma_m}e^{j\theta_m[i]}\right|^2, \tag{7}$$

where individual received SNRs $\{\gamma_m\}_{m=1}^{M}$ are known beforehand, and remain constant during the whole communication process. Since we want to analyze the effect of the amount of feedback signaling (i.e., N) based on the performance measure presented in (6), a suitable expression for the *cumulative distribution function* (CDF) $F(\Gamma[i]|\gamma_1,\ldots,\gamma_M)$ should be obtained. Unfortunately, a tractable closed-form expression for this distribution can only be obtained for very specific situations (i.e., not for all M and N). However, since in this paper we are interested in studying the outage probability when the number of active RNs is high (i.e., when $M \geq 10$), we will use the central limit theorem to show that RV (7) can be successfully approximated as the sum of two independent *chi-squared* (χ^2) distributed RVs (one central and one non-central) with 1 degree of freedom each.

A. Central and Non-Central Chi-Square Distributions

Let $\{X_k\}_{k=1}^{n}$ be independent Gaussian RVs with common variance σ^2 and non-negative mean μ_k. Then, sum

$$Y = \sum_{k=1}^{n} {X_k}^2 \tag{8}$$

follows the *non-central* χ^2 distribution with n degrees of freedom [12]. The corresponding *probability distribution function* (PDF) expression in this situation is given by

$$f_Y(y)=\frac{1}{2\sigma^2}\left(\frac{y}{s^2}\right)^{\frac{n-2}{4}}\exp\left(-\frac{s^2+y}{2\sigma^2}\right)I_{\frac{n}{2}-1}\left(\frac{s}{\sigma_2}\sqrt{y}\right) \quad y\geq 0, \tag{9}$$

where

$$s^2=\sum_{k=1}^{n}\mu_k^2 \tag{10}$$

is the non-centrality parameter of the distribution, and

$$I_\alpha(x)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos(\alpha\theta)\exp\left(x\cos\theta\right)d\theta \tag{11}$$

is the α-th order modified Bessel function of the first kind [13]. The characteristic function is also defined in closed-form, and it is given by

$$\Psi_Y(\omega)=\left(\frac{1}{1-2j\omega\sigma^2}\right)^{\frac{n}{2}}\exp\left(\frac{j\omega s^2}{1-2j\omega\sigma^2}\right). \tag{12}$$

We note that in the particular case when all means are zero (i.e., when $\mu_k=0$ for $k=1,\ldots,n$), the distribution of RV (8) reduces to the so-called *central* χ^2 distribution, whose PDF expression for n degrees of freedom is given by

$$f_Y(y)=\frac{1}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)\sigma^n}y^{\frac{n}{2}-1}\exp\left(-\frac{y}{2\sigma^2}\right) \qquad y\geq 0, \tag{13}$$

where

$$\Gamma(y)=\int_0^\infty t^{y-1}\exp\left(-t\right)dt \tag{14}$$

represents the Gamma function [12]. The characteristic function in this situation is given by

$$\Psi_Y(\omega)=\left(\frac{1}{1-2j\omega\sigma^2}\right)^{\frac{n}{2}}. \tag{15}$$

Let us now assume that

$$Z=Y_1+Y_2 \tag{16}$$

is the combination two independent χ^2 RVs: a non-central χ^2 RV with non-centrality parameter ${s_1}^2$ and variance ${\sigma_1}^2$, and a central χ^2 RV with variance ${\sigma_2}^2$, respectively. Let us also consider that the degrees of freedom are also equal in both cases (i.e., $n_1=n_2=n$). A typical way to obtain the distribution of RV Z is to calculate its characteristic function [14], i.e.,

$$\Psi_Z(\omega)=\left[\frac{1}{\left(1-2j\omega{\sigma_1}^2\right)\left(1-2j\omega{\sigma_2}^2\right)}\right]^{\frac{n}{2}}\exp\left(\frac{j\omega{s_1}^2}{1-2j\omega{\sigma_1}^2}\right). \tag{17}$$

This characteristic function can be inverse-Fourier transformed to yield the PDF

$$\begin{aligned} f_Z(z) &= \frac{1}{2{\sigma_1}^2}\left(\frac{\sigma_1}{\sigma_2}\right)^n\left(\frac{z}{{s_1}^2}\right)^{\frac{n-1}{2}}\exp\left(-\frac{z+{s_1}^2}{2{\sigma_1}^2}\right) \\ &\times \left[\sum_{k=0}^{\infty}\frac{\Gamma\left(\frac{n}{2}+k\right)}{k!\Gamma\left(\frac{n}{2}\right)}\left(\frac{\sqrt{z}\left({\sigma_2}^2-{\sigma_1}^2\right)}{s_1{\sigma_2}^2}\right)^k\right. \\ &\times \left. I_{n+k-1}\left(\frac{\sqrt{z}s_1}{{\sigma_1}^2}\right)\right] \qquad z\geq 0. \end{aligned} \tag{18}$$

It is also possible to show that the CDF in this situation admit the form

$$\begin{aligned} F_Z(z) &= \left(\frac{\sigma_1}{\sigma_2}\right)^n\sum_{k=0}^{\infty}\frac{\Gamma\left(\frac{n}{2}+k\right)}{k!\Gamma\left(\frac{n}{2}\right)}\left(\frac{{\sigma_2}^2-{\sigma_1}^2}{{\sigma_2}^2}\right)^k \\ &\times \left[1-Q_{n+k}\left(\frac{s_1}{\sigma_1},\frac{\sqrt{z}}{\sigma_1}\right)\right] \qquad z\geq 0, \end{aligned} \tag{19}$$

where

$$Q_m(a,b)=\int_b^\infty x\left(\frac{x}{a}\right)^{m-1}\exp\left(-\frac{x^2+a^2}{2}\right)I_{m-1}\left(ax\right)dx \tag{20}$$

is the generalized m-th order Marcum Q function [12].

B. Probability Distribution Approximation for Received SNR

Due to the Euler's formula, the RV

$$H[i]=\widetilde{X}_R[i]+j\widetilde{X}_I[i] \tag{21}$$

can be written in terms of its real and imaginary parts:

$$\widetilde{X}_R[i] = \sqrt{\frac{P_t}{M}}\sum_{m=1}^{M}\sqrt{\gamma_m}\cos\theta_m[i], \tag{22}$$

$$\widetilde{X}_I[i] = \sqrt{\frac{P_t}{M}}\sum_{m=1}^{M}\sqrt{\gamma_m}\sin\theta_m[i]. \tag{23}$$

Based on the fact that M is *large*, we use the central limit theorem to claim that both, real and imaginary parts of $H[i]$ are Gaussian with means μ_R and μ_I, respectively [14]. Since the imaginary part of $H[i]$ is a sum of sine functions with symmetrically distributed phases, its mean equals zero[2]. Based on the discussion presented in Section III-A, we observe that it is possible to approximate the stochastic behavior of RV $|\widetilde{X}_I[i]|^2$ as a central χ^2 distribution with 1 degree of freedom. Similarly, it is possible to see that the expected value of the real part of $H[i]$ is non-negative (actually, $\mu_R=0$ only when $N=0$). So, we claim that the stochastic behavior of RV $|\widetilde{X}_R[i]|^2$ can be approximated as a non-central χ^2 distribution with 1 degree of freedom and non-centrality parameter s_1 (unknown for the moment).

One final detail needs to be checked, to use approximation (19) for modeling the probabilistic behavior of main receiver's SNR (i.e., $\Gamma[i]$): we need to show that the real and imaginary parts of $H[i]$ (i.e., $\widetilde{X}_R[i]$ and $\widetilde{X}_I[i]$) are independent RVs. In this particular case, since $\widetilde{X}_R[i]$ and $\widetilde{X}_I[i]$ are modeled as Gaussian distributed RVs (central limit theorem), independence requirement is guaranteed if correlation coefficient between both RVs equals zero [14]. Fortunately, it is possible to show that this condition is satisfied, since the covariance of $\widetilde{X}_R[i]$ and $\widetilde{X}_I[i]$

$$C_{RI}=\mathrm{E}\{\widetilde{X}_R[i]\widetilde{X}_I[i]\}-\mathrm{E}\{\widetilde{X}_R[i]\}\,\mathrm{E}\{\widetilde{X}_I[i]\}=0 \tag{24}$$

in our system setting (detailed proof omitted).

Finally, the parameters that are required to use approximation (19) (i.e., s_1, σ_1, and σ_2) can be obtained from the first

[2]Individual phases $\theta_m[i]$ are uniformly i.i.d. on interval $(-\frac{\pi}{2^N},\frac{\pi}{2^N})$ for all m, and the sine function is an odd function.

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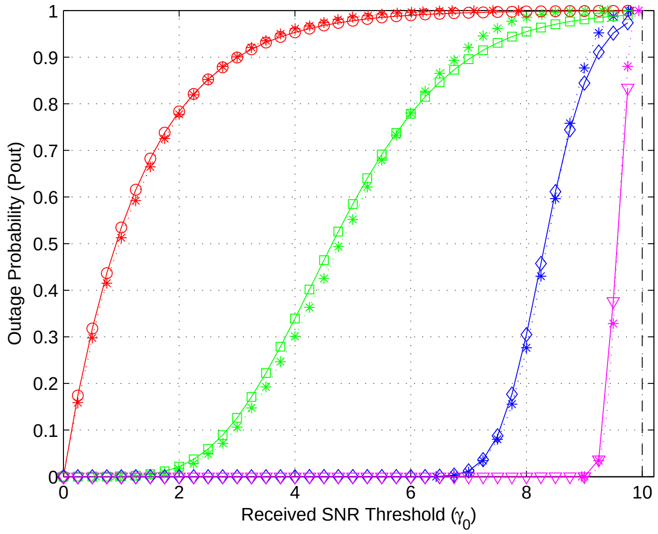

Figure 2. Outage probability as a function of SNR threshold γ_0 for DBF with 10 relays. Solid curves: No CSI ('$\circ$'), $N = 1$ ('$\square$'), $N = 2$ ('$\diamond$'), $N = 3$ ('∇'). Dashed-dotted line: Full CSI at RNs. Simulated values denoted by ('*'). Channel amplitudes are random but fixed samples from Rayleigh statistics.

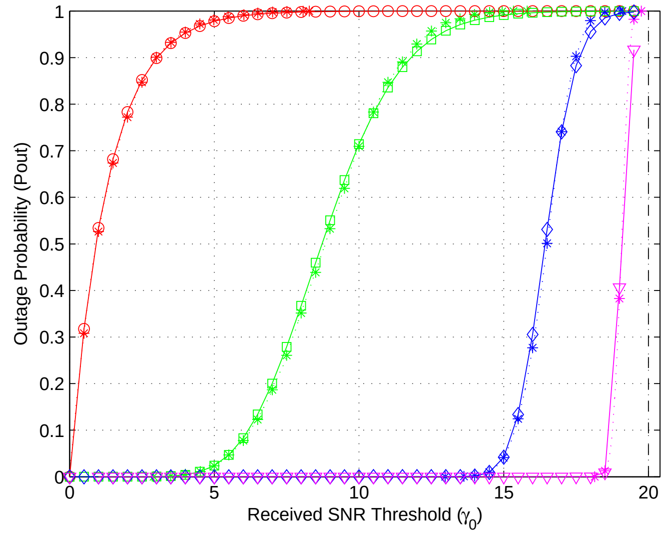

Figure 3. Outage probability as a function of SNR threshold γ_0 for DBF with 20 relays. Solid curves: No CSI ('$\circ$'), $N = 1$ ('$\square$'), $N = 2$ ('$\diamond$'), $N = 3$ ('∇'). Dashed-dotted line: Full CSI at RNs. Simulated values denoted by ('*'). Channel amplitudes are random but fixed samples from Rayleigh statistics.

two raw moments of RVs $\widetilde{X}_R[i]$ and $\widetilde{X}_I[i]$, whose closed-form expressions are obtained through simple but tedious computations:

$$\mu_R = \sqrt{\frac{P_t}{M}}\, C_N \sum_{m=1}^{M} \sqrt{\gamma_k}, \qquad \mu_I = 0, \tag{25}$$

$$\begin{aligned} \mathrm{E}\left\{\widetilde{X}_R^2\right\} &= \frac{P_t}{M}\left[\sum_{m=1}^{M} \gamma_m \left(\frac{1}{2} + \frac{1}{2} C_{N-1}\right)\right. \\ &+ \left. 2\sum_{l=1}^{M-1}\sum_{m=l+1}^{M} \sqrt{\gamma_l}\sqrt{\gamma_m}\, {C_N}^2\right], \end{aligned} \tag{26}$$

and

$$\mathrm{E}\left\{\widetilde{X}_I^2\right\} = \frac{P_t}{M}\sum_{m=1}^{M} \gamma_m \left(\frac{1}{2} - \frac{1}{2} C_{N-1}\right), \tag{27}$$

with

$$C_N = \frac{2^N}{\pi} \sin\left(\frac{\pi}{2^N}\right). \tag{28}$$

IV. Numerical Results

In this section we analyze the performance of the proposed (coherent) DBF algorithm based on the previously presented approximation. To do so we study the outage probability for different amounts of channel phase signaling (i.e., diverse N), distinct channel amplitude models (dependent on the physical location of the cooperative RNs in the system), and for various numbers of active RNs (i.e., diverse M).

Regarding to the channel amplitude models we note that in all cases, total transmission power over all relays is 0 dB and signal gains $\sqrt{\gamma_m}$ are assumed to be fixed over the whole transmission period. In addition, in those cases where RNs are grouped in two different clusters (with exactly half the number of active RNs in each one), we use notation

$$\delta = \frac{\gamma_{(1)}}{\gamma_{(2)}} \tag{29}$$

to represents the power imbalance situation between both groups. Here, $\gamma_{(1)}$ and $\gamma_{(2)}$ represent the individual SNRs of the active RNs in the first cluster (stronger channel gains) and the second cluster (weaker channel gains), respectively. We will use the following models for the channel amplitudes:

- Amplitudes are random but fixed samples from i.i.d. Rayleigh statistics.
- Amplitudes admit perfect power balance (i.e., $\delta = 0$ dB),
- Medium channel power imbalance (i.e., $\delta = 6$ dB), or
- High channel power imbalance (i.e., $\delta = 10$ dB).

When channel amplitudes are (constant) Rayleigh distributed, it is assumed that individual channel SNRs are i.i.d. exponential distributed with unitary mean value.

Figure 2 and Fig. 3 show the outage probability for a SNR threshold when using the proposed DBF scheme for different amounts of channel phase signaling in case of $M = 10$ and $M = 20$ active RNs, respectively. In this scenario (constant) Rayleigh distributed channel amplitudes were used to model the amplitudes. The solid curves are plotted based on approximation (19) with appropriate fitting parameters, along with asymptotic upper bounds in case of full CSI at RNs (dashed line) [3]. In all cases, simulated point values ('$*$') are also included to verify the validation of the analytical results. Based on the results it is observed that our approximation follows simulated values well. As expected, the accuracy of

[3]Full CSI is actually a synonym of perfect channel phase information because no channel amplitude information is considered in this work.

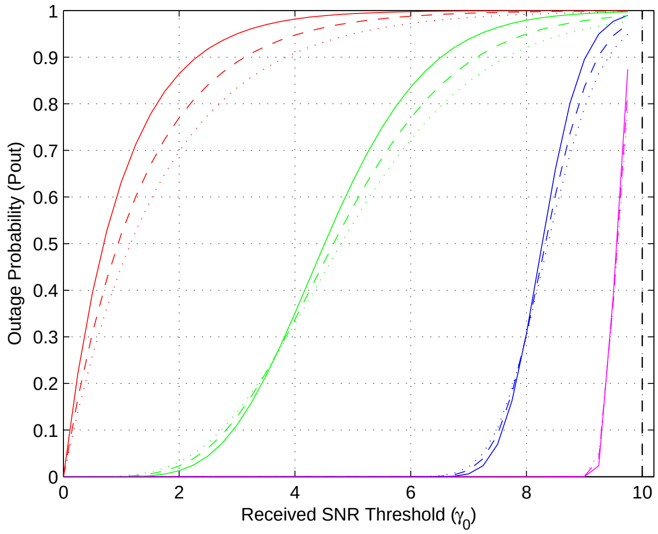

Figure 4. Outage probability as a function of SNR threshold γ_0 for DBF with 10 relays. Solid curves: Perfect channel balance (i.e., $\delta = 0$ dB). Dashed lines: Medium channel imbalance (i.e., $\delta = 6$ dB). Dotted curves: Large channel imbalance (i.e., $\delta = 10$ dB). Channel feedback: No CSI (red), $N = 1$ (green), $N = 2$ (blue), $N = 3$ (magenta). Dashed-dotted lines: Full CSI at RNs.

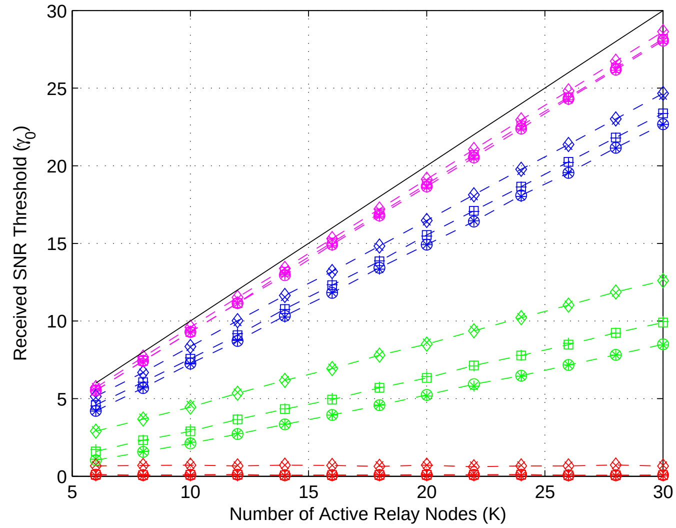

Figure 6. Required SNR threshold γ_0 for DBF to guarantee a given outage probability as a function of number active relays. Outage probability: $\Pr_{\text{out}} = 0.02$ ('○'), $\Pr_{\text{out}} = 0.1$ ('□'), $\Pr_{\text{out}} = 0.5$ ('◇'). Channel feedback: No CSI (red), $N = 1$ (green), $N = 2$ (blue), $N = 3$ (magenta). Solid line: Full CSI at RNs. Simulated values denoted by ('*'). Channel amplitudes are fixed with perfect power balance.

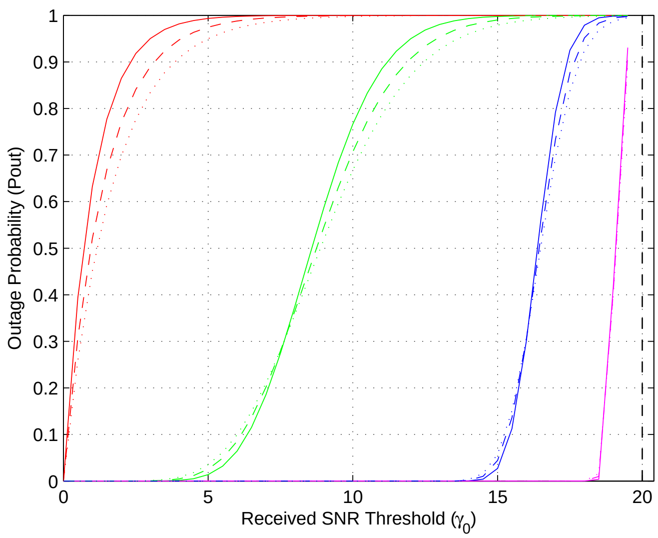

Figure 5. Outage probability as a function of SNR threshold γ_0 for DBF with 20 relays. Solid curves: Perfect channel balance (i.e., $\delta = 0$ dB). Dashed lines: Medium channel imbalance (i.e., $\delta = 6$ dB). Dotted curves: Large channel imbalance (i.e., $\delta = 10$ dB). Channel feedback: No CSI (red), $N = 1$ (green), $N = 2$ (blue), $N = 3$ (magenta). Dashed-dotted lines: Full CSI at RNs.

the approximation is better when the number of active RNs in the system is higher. The outage probability in absence of channel phase signaling is used as a baseline. It is found that performance in terms of outage probability clearly increases with additional phase bits in the feedback link. We also note that if $N = 3$, then the performance of DBF scheme is very close to the one observed with full CSI at RNs.

Figure 4 and Fig. 5 show the outage probability for given SNR threshold when implementing DBF algorithm in different channel power imbalance situations. In this case RNs are grouped in two clusters (of the same size), that are located at different distances from the main receiver. Solid curves, dashed curves, and dotted curves represent perfect channel power balance (i.e., $\delta = 0$ dB), medium channel power imbalance (i.e., $\delta = 6$ dB), and high channel power imbalance (i.e., $\delta = 10$ dB) situations, respectively. Based on the results we observe that, the power imbalance level in the channel amplitude model increases the outage probability of DBF algorithm for low SNR thresholds. The larger is the number of phase bits N, the smaller is this impairment. The same behavior is visible when the number of active RN increases. This is because of the increasing variability that main receiver faces in its SNR in both situations, causing a less abrupt improvement on CDF curve as the value of γ_0 grows.

Finally, Fig. 6 presents the the maximum SNR threshold that can be guaranteed for a given outage probability when implementing our DBF algorithm in a perfect channel power balance case (i.e., $\delta = 0$ dB). These curves admit almost linear behavior with respect to the number of active RNs. Based on these curves we observe that, as N grows, the gap between the different outage probability curves decreases. This is in accordance with the behavior of the expected value of the real part of the sum channel (i.e., μ_R), given in equation (25) and presented in Fig. 7.

In the light of all results we see that there is no reason to use more than $N = 3$ bits for phase feedback per RN. Yet, the performance that is obtained with $N = 1$ bit is not good enough. However, the performance obtained with $N = 2$ bits represents a reasonable tradeoff between *the cost* of signaling overhead, and *the benefit* of the outage probability performance improvement that is observed.

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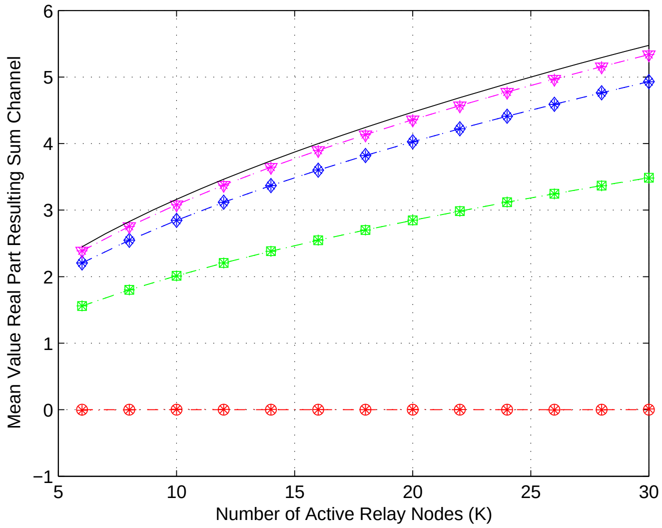

Figure 7. Expected value for the real part of the sum channel μ_R as a function of number active relays. Dashed curves: No CSI ('∘'), $N = 1$ ('□'), $N = 2$ ('◇'), $N = 3$ ('▽'). Solid line: Full CSI at RNs. Simulated values denoted by ('*'). Channel amplitudes are fixed with perfect power balance.

V. Conclusion

In this paper we studied the performance of a *distributed beamforming* (DBF) algorithm in presence of different amounts of channel phase feedback information. This analysis is done in the context of wireless system, where the subscriber (main transmitter) receives assistance from a cooperative network to boost its communication to base station (main receiver). This cooperative network is formed by a large number of low-cost *relay nodes* (RNs), deployed in the premises of the subscriber. Location of the RNs are assumed to be fixed during the whole duration of the data transmission. Due to short distances, the communication over the first hop (i.e., from main transmitter to RNs) is assumed to be cheap in terms of transmission power and radio resource usage. Therefore, the bottleneck lies in the second hop (i.e., from RNs to the main receiver).

The outage probability for a given target *signal-to-noise* power *ratio* (SNR) is used as performance measure. To carry out the analysis, a suitable closed-form approximation for *cumulative distribution function* (CDF) of received SNR is derived. The parameters for this CDF approximation are obtained from the first two raw moments of the resulting sum channel that main receiver observes. The derived CDF expression is validated using simulations. Our analysis reveals that the use of DBF with a small amount of phase feedback information allows to reap a large fraction of the power gain that is available in the second hop.

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