Electrical Equivalent Modeling of Single Component Fractional Order Element with Porous Surface

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Abstract—**A single component fractional order element (FOE) is indigenously developed and its electrical equivalent model is proposed. The FOE exhibits fractional order behavior due to anomalous diffusion of ions through the porous surface of the electrode. The model includes the effect of the polarizable solution and diameter of the pores on the parameters of the FOE. The simulation results of the proposed electrical equivalent model obtained by changing its parameters are discussed along with the experimental results illustrating the behavior of single component FOE.**

Keywords-Single component fractional order element (FOE); anomalous diffusion; porous electrode.

I. INTRODUCTION

Design and development of fractional order element (FOE) has become an important field of research owing to the growing interest in studying fractional order systems. The earlier technique for realization of a fractional order element was using different combinations of RC ladder networks [1], [2], [3], [4]. But recently some authors have reported realization of FOE by using physico-chemical phenomenon [5]. A single component fractional order element has been developed by the authors indigenously [6], [7], [8], where a capacitor type probe with porous surface behave as an FOE when dipped in ionic medium.

To understand the exact relationship among the fractional exponent of FOE and the fabrication parameters, the mechanism due to which the fractional order behavior is exhibited must be studied. Literature shows that anomalous diffusion of ions [9], [10] through fractal surface results in fractional order behavior. In case of the single component FOE, the porous PMMA coating on the electrode surface consists of self similar spherical pores. Thus the electrode surface is fractal in nature, ion diffusion through which results in the fractional behavior.

It is well known that using different circuit combinations of resistance (R) and capacitance (C), i.e., cross RC ladder network, domino ladder, tree structure, an FOE can be realized. Thus a single component FOE can also be modeled using some combinations of R and C.

In this paper, the relation between the parameters of single component FOE and the fabrication criteria has been identified. Also the effect of diffusion of ions through the porous electrode surface on the behavior of the FOE is explored. An electrical equivalent model is proposed which is required for designing the single component FOE with desired specifications.

The paper is organised in four sections. Section II deals with the basics of single component fractional order element and the proposed electrical equivalent model for it. In Section III the experimental results showing the behavior of single component FOE as well as the simulation results obtained from the electrical equivalent model are discussed. Finally, in Section IV, concluding remarks and future scope have been looked into.

II. ELECTRICAL EQUIVALENT MODEL OF SINGLE COMPONENT FOE

A single component fractional order element is fabricated by coating a copper plated epoxy glass with a porous film of poly-methyl-methacrylate (PMMA). This probe is dipped in a polarizable medium. The polymer coated copper probe when dipped in polarizable solution behaves as a fractional order element. The diagram of single component FOE is shown in Figure 1.
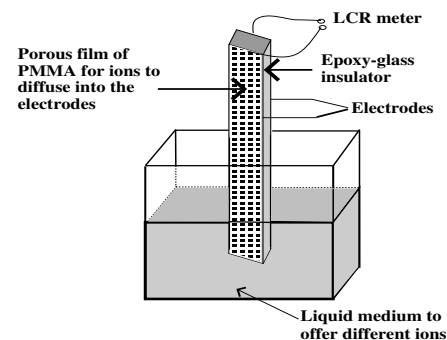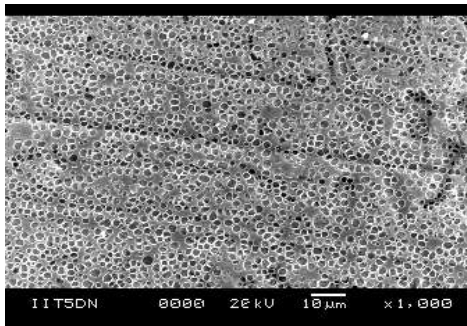

Figure 1. Diagram of single component FOE

Figure 2. SEM image of the electrode surface of single component FOE with coating thickness $5\mu m$; Pore diameter = $0.85\mu m$
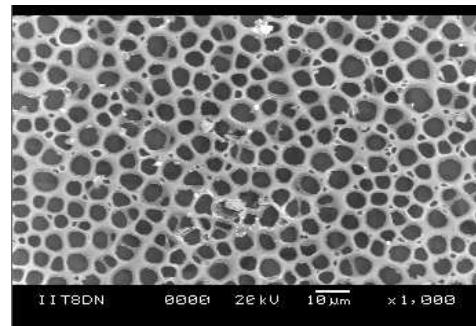

Figure 4. SEM image of the electrode surface of single component FOE with coating thickness $15\mu m$; Pore diameter = $2.24\mu m$
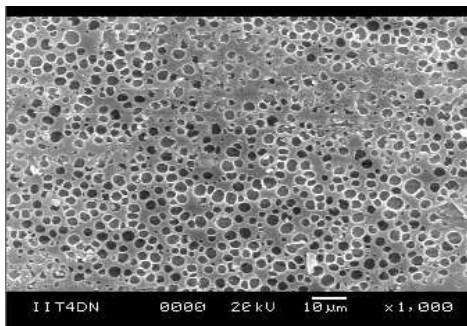

Figure 3. SEM image of the electrode surface of single component FOE with coating thickness $10\mu m$; Pore diameter = $1.27\mu m$
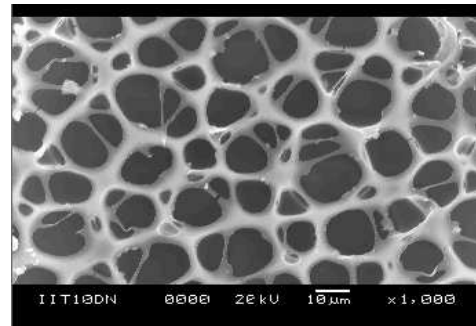

Figure 5. SEM image of the electrode surface of single component FOE with coating thickness $25\mu m$; Pore diameter = $4.85\mu m$

The impedance of a fractional order element is given by

$$Z(s) = Qs^{-\alpha} \tag{1}$$

where 'α' is the fractional exponent of the FOE.

Single component FOEs having different values of 'α' can be realized by varying the following fabrication parameters
i. Thickness of PMMA coating
ii. Concentration of the polarizable solution
iii. Area of contact of the electrode surface with the solution.

The fractional behavior of the probe arises due to the anomalous diffusion of ions through the porous surface of the single component FOE. In ordinary diffusion, the mean square displacement ($\langle r^2 \rangle$) of diffusing particles is directly proportional to time (t), i.e., $\langle r^2 \rangle \propto t$. On the other hand, in case of anomalous diffusion, $\langle r^2 \rangle$ has a power law distribution on time, i.e., $\langle r^2 \rangle \propto t^{\beta}$, where $\beta < 1$. This occurs due to the distribution of time constant resulting from ion diffusion through porous surfaces.

The surface of the FOE is porous in nature and the pores are more or less circular as is evident from the Scanning Electron Microscope (SEM) images (Figures 2, 3, 4, 5).

It can be seen from Figures 2, 3, 4, 5 that the pore diameter has an one to one relationship with the thickness of the PMMA film, i.e., the pore size increases with increase in thickness of the PMMA coating and vice versa.

The pores on the electrode surface are assumed to be spherical. At different layers of the film, the pore diameter is different. As the ions penetrate through the porous film, the resistance offered by the solution increases due to decrease in pore size (Figures 2, 3, 4, 5). The ions continue to penetrate till the pore diameter is less than the diameter of the ions. At this point the ions cannot move any further and become stationary.
Let, diameter of ion be 'd_i';
Thickness of the PMMA coating be 't';
Pore diameter corresponding to coating thickness 't' be 'D';
Number of branches the ions penetrate be 'n' and
Ratio of decrease of the pore diameter between two successive layers be 'a'.

The ions can penetrate till the pore diameter is less than 'd_i'. This can be mathematically represented by

$$\frac{D}{a^n} \le d_i \tag{2}$$

The movement of the ions through the porous film can be modeled using a tree network consisting of resistances and capacitances. The electrode surface consists of numerous pores. Considering a single pore, the cross sectional view of the film can be represented as shown in Figure 6.

The porous structures are considered to be self similar in nature, i.e., from each pore 'N' number of smaller pores
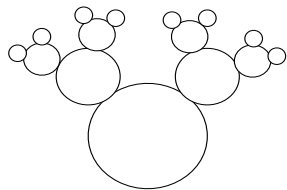
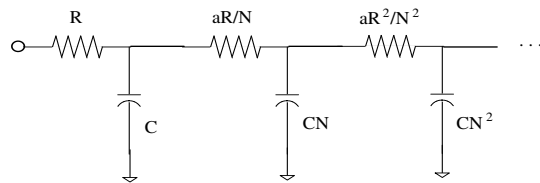Figure 6.   Self similar porous structure on the electrode surface

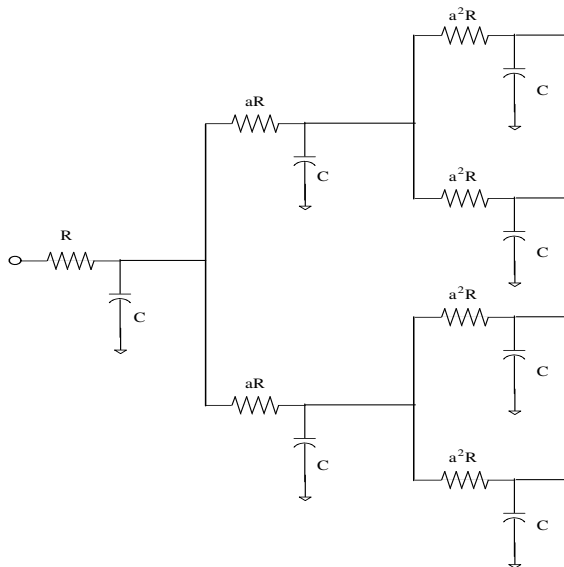Figure 8.   Equivalent ladder network of the porous electrode surface

Figure 7.   Equivalent tree network of the porous electrode surface

Figure 9.   Simulation results showing change in 'α' with change in ratios 'g' and 'G'

branch out and this structure continues. Each spherical pore can be represented by a simple series RC circuit. The equivalent circuit is shown in Figure 7.

The resistance value depends on the resistivity (ρ) of the polarizable solution and the surface area of the pore.

$$R = \frac{\rho D}{4\pi(\frac{D^2}{4})} = \frac{\rho}{\pi D} \qquad (3)$$

The capacitance C is the interfacial capacitance between two layers of the film.

$$C = \frac{4\pi(\frac{D^2}{4})\epsilon}{D} = \pi\epsilon D \qquad (4)$$

where, ϵ is the permittivity of PMMA. The impedance of the circuit shown in Figure 7 can be represented by [11]

$$Z_n = R + \frac{1}{j\omega C +} \frac{2}{aR +} \frac{1}{j\omega C +} \frac{2}{a^2 R} \cdots \qquad (5)$$

The tree form of Figure 7 is equivalent to the ladder network [12] as shown in Figure 8.

In the above circuit, 'N' is the number of sub branches from each node.
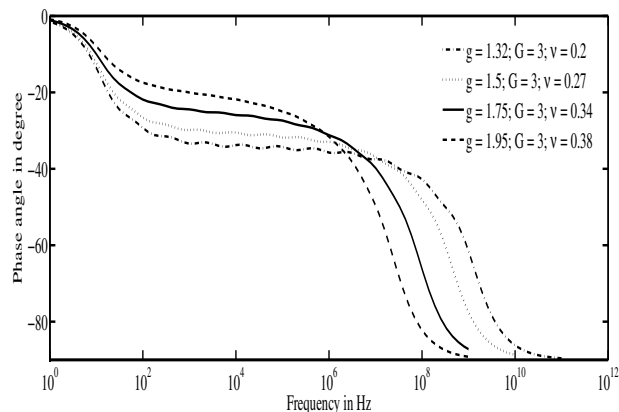Geometric ratio of resistance $(g) = a/N$

Geometric ratio of capacitance $(G) = N$
Parameter 'ν' is such that [3], [13]

$$\nu = \frac{lnG}{ln(Gg)} = \frac{lnN}{lna} \qquad (6)$$

Thus, the fractional exponent of the ladder network is given by

$$\alpha = 1 - \nu = 1 - \frac{lnG}{ln(Gg)} = 1 - \frac{lnN}{lna} \qquad (7)$$

'ν' can be defined as the fractal dimension of the single component FOE. For the model to behave as a fractional order element, the value of 'N' should be greater than 1.

III.   INFERENCES FROM THE ELECTRICAL EQUIVALENT MODEL

Simulation of the ladder network given in Figure 8 shows that the value of 'α' depends on the ratios 'g' and 'G' (Figure 9).

It has been observed that by changing the product of R and C the bandwidth of constant phase angle can be shifted. On increasing the product RC the bandwidth shifts to lower frequency range and vice versa. This phenomenon is illustrated in Figure 10.

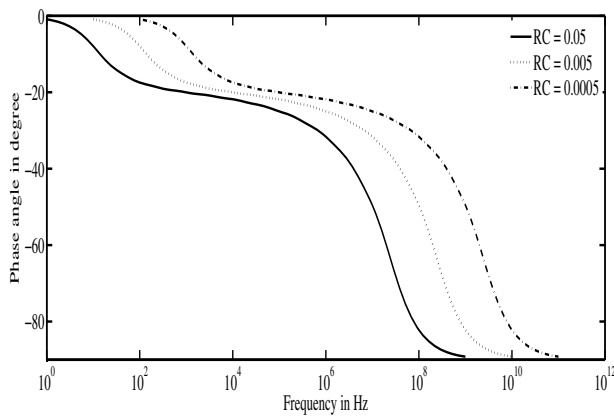Also the frequency range of constant phase angle can be varied by changing the value of 'n' (Figure 11).

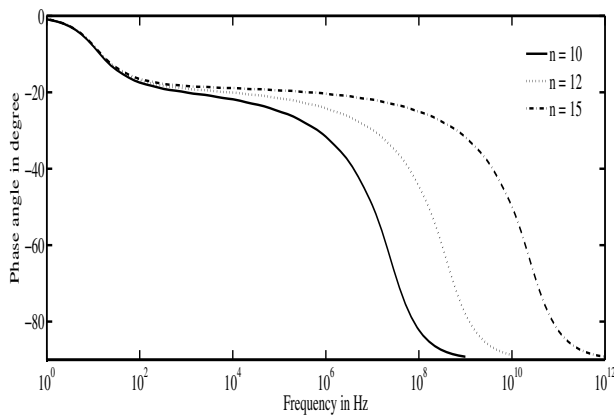Figure 10.   Simulation results showing effect of the product RC on bandwidth of constant phase angle

Figure 12.   Experimental results showing shift in bandwidth of constant phase angle and change in 'α' value with concentration change of KCl solution for FOE5

Figure 11.   Simulation results showing change in bandwidth of constant phase angle with the value of 'n'

when concentration of the polarizable solution decreases, its conductivity decreases. Hence, the resistance offered by the solution increases. The interfacial capacitance remaining constant, the product of R and C increases, thus shifting the bandwidth of constant phase angle to lower frequency range. This is evident from Figure 12.

Keeping concentration of polarizable solution fixed, i.e., 'n' constant, if different polarizable solutions are used then the solution having ions of larger radius results in smaller 'α' value as evident from (2) and Tables. II and III.

IV. CONCLUSION AND FUTURE WORK

In this work, an electrical equivalent model of the single component FOE has been proposed which takes into account the effects of anomalous diffusion of ions through porous surface and fractal dimension. The factors affecting the value of the fractional exponent 'α' and also their relationship with 'α' have been identified. The electrical equivalent model will help in predicting the behavior of single component FOE and also designing an FOE with desired specifications. The exact mathematical relation among the fabrication parameters and 'α' is yet to be established and needs further research.

From (7) it can be observed that 'α' depends on 'a' and 'N'. The value of 'α' increases with an increase in 'a'. It has been found experimentally that 'α' increases with the thickness of the PMMA coating (Table. I). Hence, 'a' is directly proportional to the PMMA film thickness. The fractional exponent, 'α' also depends on the concentration of the polarizable medium. This dependence can be incorporated in the theoretical model in terms of the value of 'n'. Experiments show that the constant phase angle of a single component FOE increases with the decrease in concentration. This can be seen from Figure 12 and Tables. II and III. Thus, we can infer that with concentration of the ionic medium, the value of 'n' increases. The increase in 'n' results in decrease in value of 'α' and increase in bandwidth of constant phase angle. From (2) it is obvious that if 'n' increases keeping 'D' and 'd_i' constant, the value of 'a' decreases and in turn 'α'. An important observation is that the bandwidth of constant phase angle shifts to lower frequency range as concentration decreases. This is because
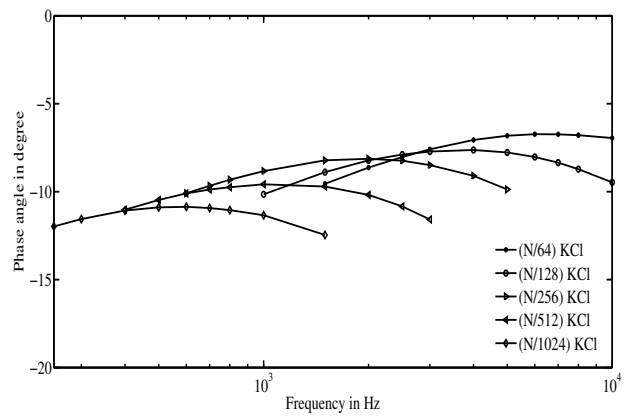
REFERENCES

[1] G. E. Carlson and C. A. Halijak, "Approximation of fractional capacitors $(1/s)^{1/n}$ by a regular newton process," *IEEE Transactions on Circuits and Systems*, vol. CAS-11, pp. 210–213, June 1964.

[2] S. C. D. Roy, "On the realization of a constant-argument immittance or fractional operator," *IEEE Transactions on Circuit Theory*, vol. CT-14, pp. 264–274, September 1967.

[3] K. B. Oldham and C. G. Zoski, "Analogue instrumentation for processing polarographic data," *Journal of Electroanalytical Chemistry*, vol. 157, pp. 27–51, 1983.

Table I
EXPERIMENTAL RESULTS SHOWING THE CHANGE OF 'α' WITH PMMA COATING THICKNESS

Name	Thickness (μm)	Pore size (μm)	Polarizable medium	Conductivity (mS/cm)	Q	α	Frequency range
FOE1	10	1.27	pH7.89	0.24	20.34×10^4	0.31	$100\ Hz - 100\ kHz$
FOE2	15	2.24	pH7.89	0.24	46.28×10^4	0.32	$100\ Hz - 4\ kHz$
FOE3	45	12.37	pH7.89	0.24	73.08×10^6	0.66	$200\ kHz - 1\ MHz$
FOE4	50	14.72	pH7.89	0.24	46.41×10^7	0.76	$200\ kHz - 1\ MHz$

Table II
EXPERIMENTAL RESULTS SHOWING THE EFFECT OF CONCENTRATION OF POLARIZABLE SOLUTION (KCl) ON 'α' AND BANDWIDTH OF CONSTANT PHASE ANGLE; IONIC RADIUS OF K = 1.38 $Angstrom$

Name	Thickness (μm)	Pore size (μm)	Polarizable medium	Conductivity (mS/cm)	Q	α	Frequency range of constant phase angle	Bandwidth (decade)
FOE5	12	1.64	(N/64) KCl	2.7	7.07	0.084	$1.5\ kHz - 20\ kHz$	1.12
FOE5	12	1.64	(N/128) KCl	1.52	16.51	0.094	$1\ kHz - 10\ kHz$	1
FOE5	12	1.64	(N/256) KCl	0.85	34.24	0.099	$600\ Hz - 5\ kHz$	0.92
FOE5	12	1.64	(N/512) KCl	0.48	75.02	0.115	$400\ Hz - 3\ kHz$	0.875
FOE5	12	1.64	(N/1024) KCl	0.27	136.92	0.127	$250\ Hz - 1.5\ kHz$	0.78

Table III
EXPERIMENTAL RESULTS SHOWING THE EFFECT OF CONCENTRATION OF POLARIZABLE SOLUTION ($NaCl$) ON 'α' AND BANDWIDTH OF CONSTANT PHASE ANGLE; IONIC RADIUS OF Na = 1.02 $Angstrom$

Name	Thickness (μm)	Pore size (μm)	Polarizable medium	Conductivity (mS/cm)	Q	α	Frequency range of constant phase angle	Bandwidth (decade)
FOE5	12	1.64	(N/32) NaCl	3.4	13.26	0.097	$800\ Hz - 10\ kHz$	1.097
FOE5	12	1.64	(N/64) NaCl	2.1	25.78	0.099	$500\ Hz - 6\ kHz$	1.079
FOE5	12	1.64	(N/128) NaCl	0.92	68.51	0.115	$200\ Hz - 2\ kHz$	1
FOE5	12	1.64	(N/256) NaCl	0.59	197.05	0.138	$100\ Hz - 800\ Hz$	0.9

[4] M. Sugi, Y. Hirano, Y. Miura, and K. Saito, "Frequency behavior of self-similar ladder circuits," *Physicochemical and Engineering Aspects*, pp. 198–200, 2002.

[5] G. W. Bohannan, "Application of fractional calculus to polarization dynamics in solid dielectrics materials," in *PhD dissertion*, 2000, Montana, State University, Bozeman.

[6] K. Biswas, S. Sen, and P. K. Dutta, "Realization of a constant phase element and its performance study in a differentiator circuit," *IEEE Transactions on Circuits and Systems-II, Express Briefs*, vol. 53, pp. 802–806, September 2006.

[7] K. Biswas, *Studies on Design, Development and Performance Analysis of Capacitive Type Sensors*. PhD thesis, Indian Institute of Technology Kharagpur, India, Department of Electrical Engineering, February 2007.

[8] M. Sivaramakrishna, S. Das, K. Biswas, and B. Goswami, "Characterization of a fractional order element realized by dipping a capacitive type probe in polarizable medium," in *Symposium on Fractional Signals and Systems Lisbon09*, November 2009.

[9] J. Bisquert and A. Compte, "Theory of the electrochemical impedance of anomalous diffusion," *Journal of Electroanalytical Chemistry*, vol. 499, pp. 112–120, 2001.

[10] A. Lasia, "Electrochemical impedance spectroscopy and its applications," *Modern Aspects of Electrochemistry*, vol. 32, pp. 143–248, 1999.

[11] S. H. Liu, "Fractal model for the ac response of a rough surface," *Physical Review Letters*, vol. 55, no. 5, pp. 529–532, 29 July, 1985.

[12] B. Sapoval, J. N. Chazalviel, and J. Peyrière, "Electrical response of fractal and porous interfaces," *Physical Review A*, vol. 38, no. 11, pp. 5867–5887, December 1, 1988.

[13] M. Moshrefi-Torbati and J. K. Hammond, "Physical and geometrical interpretation of fractional operators," *J. Franklin Inst.*, vol. 335B, no. 6, pp. 1077–1086, 1998.

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