

# A Rapid Modeling and Prototyping Technique for Piezoelectric Energy Harvesting Systems

Aldo Romani, Enrico Sangiorgi, Marco Tartagni  
 Advanced Research Center on Electronic Systems  
 University of Bologna – Cesena Campus  
 Cesena, Italy  
 {aldo.romani, e.sangiorgi, marco.tartagni}@unibo.it

Rudi P. Paganelli  
 IEIIT-CNR  
 National Research Council (CNR)  
 Bologna, Italy  
 rudipaolo.paganelli@cnr.it

**Abstract**—This paper describes a reliable modeling technique for piezoelectric transducers and a procedure for identifying model parameters with few simple measurements and standard laboratory equipment. Direct measurements were taken on commercial Q220-A4-303YB piezoelectric transducers from Piezo Systems in a cantilever configuration with tip masses between 6 g and 18 g. For validation purposes, the behavior of the equivalent electromechanical circuits was simulated and compared to direct observations in real operating conditions. The model showed to predict with a high degree of accuracy both the response of the transducers to vibrations and a set of second order effects associated to the presence of a non-linear power converter and usually ignored by conventional models presented in literature.

**Keywords:** piezoelectric devices, energy harvesting, model identification, equivalent electromechanical circuit

## I. INTRODUCTION

Piezoelectric transducers are widely used in energy harvesting applications. Small-sized oscillating structures with seismic masses attached are often used (Fig. 1a). Several types of electronic interfaces have also been proposed up to now for collecting electrical charge from the transducers [1].

The behavior of this kind of systems depends on many factors such as the input vibrations, the shape of the transducer, the seismic mass attached to the transducer and the electronic interface. During the design phase a rapid and reliable quantitative estimate of the behavior of transducers and circuits is highly desirable for optimizing the system as a whole. In this task two main difficulties coexist: (a) in piezoelectric transducers electrical and mechanical quantities are strictly coupled; (b) conversion circuits are usually non-linear and include power transistors, diodes and digital

control. Currently, especially during the design phase, there is still need of reliable methods for rapid prototyping and predicting the behavior of such complex systems.

An analytical approach [2] can accurately model the behavior of the piezoelectric transducer itself, but such models cannot be used in circuit simulations. In [3] a modeling approach based on an equivalent electromechanical circuit with lumped parameters provided reliable results. This type of approach is also suitable for joint simulations with non linear (e.g. switching) power converter circuits. However, in practical cases not all the required electrical, mechanical and geometrical parameters are known. Nonetheless, an accurate modeling is essential for carrying out system design efficiently. In fact, the way charge is extracted and transferred away from the transducer might significantly affect the response to vibrations of the transducer itself. Recent works on non-linear power conversion circuits [1][4][5] made use of assumptions such as weak electromechanical couplings, forced displacements or purely capacitive transducers (Fig. 1b) which are not valid in many practical cases, especially when the transducers operate near resonance.

This paper presents a new technique for a fast and reliable identification of the parameters of the equivalent electromechanical circuit of a generic piezoelectric transducer. The proposed approach is general and the technique can be applied to any type of transducer. For validation purposes, the equivalent circuits of a set of commercial piezoelectric transducers will be identified and a series of experimental measurements in both frequency and time domain will be compared to simulations. The results will show that the modeling technique produces reliable results in predicting the behavior of the transducers.

## II. EQUIVALENT ELECTROMECHANICAL CIRCUIT OF A PIEZOELECTRIC TRANSDUCER

As shown in Fig. 1, in energy harvesting applications piezoelectric transducers are usually coupled with a seismic mass in order to build up a mechanical oscillator. Because of formal analogies between Newton’s laws and Kirchoff’s laws, as discussed in [3], an equivalent electromechanical circuit such as the one shown in Fig. 2a can model the behavior of a piezoelectric transducer .

The mechanical part may be described in terms of forces ( $F$ ) and velocities ( $\dot{z}$ ), where  $z$  is the deflection of the cantilever.

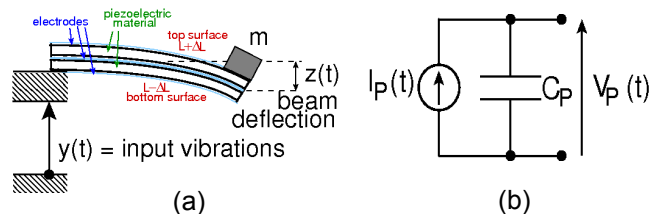


Figure 1. (a) Structure of a piezoelectric cantilever beam with a tip mass harvesting energy from vibrations; (b) a simplified equivalent circuit of a piezoelectric transducer.

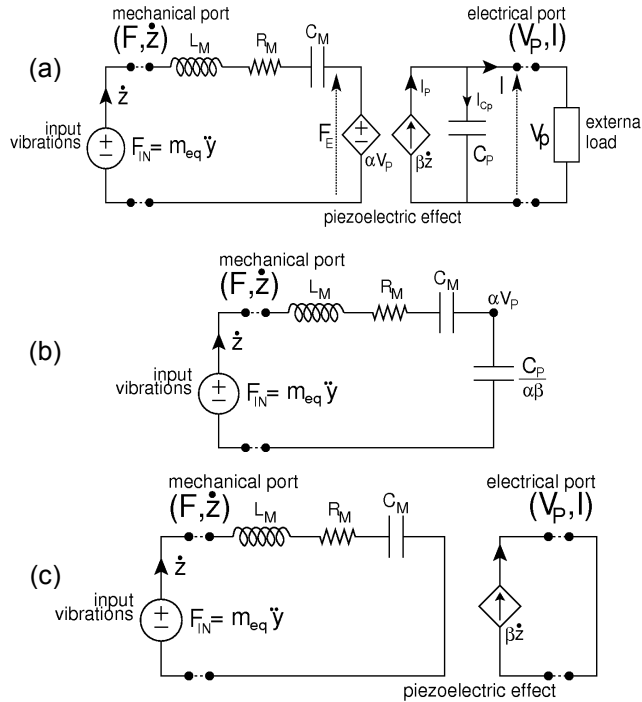


Figure 2. (a) generic equivalent electromechanical circuit of a piezoelectric transducer modeled as a two-port network; (b) equivalent oscillator circuit of a piezoelectric transducer in open circuit configuration; (c) equivalent oscillator circuit of a piezoelectric transducer in a short circuit configuration

In general, according to the laws of dynamics it holds that  $F_{IN} = m_{eq} \ddot{y}$ , where  $F_{IN}$  is the force acting on the transducer (e.g. expressed in N),  $\ddot{y}$  is the amplitude of input vibrations (e.g. in  $m/s^2$ ) and  $m_{eq}$  the equivalent mass of the oscillating system (e.g. in kg) which is related to the inertial mass and to the geometries of the transducer. In this paper, without losing generality, all the forces acting on the system will be expressed in scaled units by assuming a unity equivalent mass  $m_{eq}$ . This notation implies that the input force  $F_{IN}$  is considered numerically equal to the amplitude of input vibrations  $\ddot{y}$ , which can be directly measured. On the contrary, an exact value of the equivalent mass  $m_{eq}$  cannot be easily determined with direct measurements.

Each lumped element of the equivalent circuit takes into account different physical quantities:  $L_M$ ,  $R_M$ ,  $C_M$  are respectively related through  $(F, \dot{z})$  to kinetic energy, mechanical losses and elastic energy of the system;  $C_P$  is the electrical capacitance measured between the surface electrodes of the piezoelectric element; the coefficients  $\alpha$ ,  $\beta$  respectively model the inverse and the direct piezoelectric effects. Additional vibration modes can be taken into account by including additional mechanical resonators.

An equivalent circuit approach, provided that a reliable estimate of its parameters is provided, allows to perform accurate simulations of the transducer connected to any power conversion circuit or external load. In next section a parameter identification technique based on the observation of few significant quantities will be presented.

### III. IDENTIFICATION OF MODEL PARAMETERS

In general, as shown in Fig. 2b, the piezoelectric transducer in an open circuit configuration basically behaves as an oscillator with a resonance frequency  $f_0$ :

$$f_0 = \sqrt{\frac{1}{L_M C_{EQ}} - \frac{1}{\tau_M^2}}, \quad (1)$$

where  $C_{EQ} = C_M C_P / (C_P + \alpha \beta C_M)$  and  $\tau_M$  is the exponential damping time constant defined below.

When the terminals of the transducer are shorted ( $V_P = 0$ ), as shown in Fig. 2c, a pure mechanical oscillator is built up with time constant  $\tau_M$  and resonance frequency  $f_M$ :

$$\tau_M = \frac{2L_M}{R_M}, \quad f_M = \sqrt{\frac{1}{L_M C_M} - \frac{1}{\tau_M^2}}. \quad (2)$$

It can be demonstrated that the transducer can be considered shorted whenever we connect to its terminals a resistor  $R_L$  whose value is low enough to drain almost all the current  $\beta \dot{z}$  coming from the controlled current source, so that the effect of the  $\alpha V_P$  voltage generator on the mechanical part is cancelled. When  $R_L \cong 0$  the following relations can be determined by solving circuit equations:

$$\beta = \frac{V_P(f_M)}{R_L \cdot \pi f_M \cdot \Delta z_M} \quad (3)$$

$$L_M = \frac{F_{IN}(f_M)}{\Delta z_M} 2 \tau_M^2 \sqrt{\frac{1}{1 + 16(\pi f_M)^2 \tau_M^2}} \quad (4)$$

$$\frac{1}{C_M} = L_M \left[ (2\pi f_M)^2 + 1/\tau_M^2 \right] \quad (5)$$

where  $\Delta z_M$  is the peak-to-peak displacement of the shorted transducer at the mechanical resonance  $f_M$ .

Conversely, when the transducer is in open circuit configuration the following equation holds:

$$\alpha = \frac{1}{\beta} \frac{C_P}{C_M} \left\{ L_M C_M \left[ (2\pi f_0)^2 + 1/\tau_M^2 \right] - 1 \right\} \quad (6)$$

### IV. EXPERIMENTAL SETUP AND RESULTS

This section describes the experimental setup used for the identification of model parameters. The measurements were performed on a set of Q220-A4-303YB piezoelectric transducers from Piezo Systems with tip masses respectively of 6 g, 10 g, 12 g, 18 g. The transducers were mounted on a

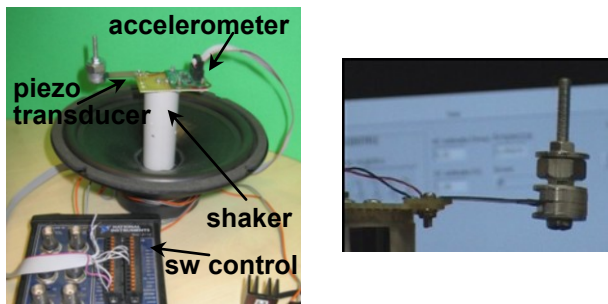


Figure 3. The cost effective shaker system composed of a standard woofer and a digital accelerometer operated under software control in a Labview environment and a detailed view of the piezoelectric transducers used in this work.

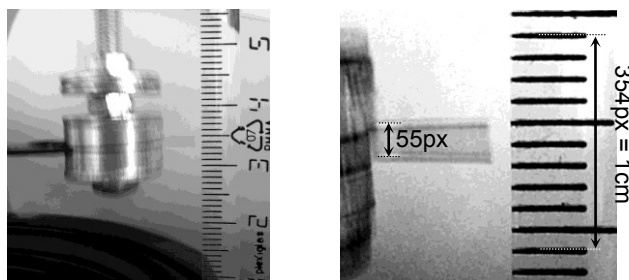


Figure 4. The peak to peak displacement can be directly measured by setting the shaker oscillating frequency and by taking a shot with 1” exposure time with a standard digital camera. Pixel resolution allows to measure the displacement with satisfactory accuracy. As an examples, some measurements are reported on the right.

cost-effective custom shaker system based on a Ciare CW200Z woofer (Fig. 3). A Kionix KXP-84 accelerometer was also mounted on the shaker for measuring the actual input acceleration. The shaker was used for generating sinusoidal vibrations of given frequency and amplitude or other test signals. An Agilent 34401A digital multimeter and a sampling oscilloscope were used for measuring the voltages produced by the transducers. A resistor  $R_L = 98 \Omega$  was used for emulating the shorted devices, as discussed in previous section.

The identification procedure is herein summarized. First, a series of measurements is performed on the mechanical oscillator, with  $R_L$  connected as a load.

1. The shaker produces very short mechanical pulses, so that damped voltage oscillations can be observed with an oscilloscope. The damping time constant  $\tau_M$  can be easily obtained (e.g. from the screen coordinates  $(t_1, V_1)$  and  $(t_2, V_2)$  of two subsequent maxima with the expression:  $\tau_M = (t_2 - t_1) / \ln(V_1/V_2)$ .
2. The shaker generates sinusoidal accelerations at different frequencies at which the output voltage  $V_p(f)$  and the input acceleration  $\ddot{y}(f)$  are measured. The frequency at which  $V_p(f)/\ddot{y}(f)$  is maximum is identified as  $f_M$ .
3. The peak to peak displacement  $\Delta z_M$  at  $f_M$  is measured with a common digital camera. A ruler is positioned

next to the transducer and a 1” exposure shot is taken (Fig. 4). The bitmap image then is analyzed at pixel level for measuring the displacement.

Then, additional measurements are performed on the transducer in an open circuit configuration:

4. Step 2 is repeated with no load resistor in order to identify  $f_0$ ;
5. The electrical capacitance  $C_p$  of the transducer is measured with an Agilent 4284A LCR-meter, at a 10 kHz frequency well above the mechanical cut-off frequency, so as to neglect all mechanical effects.

Finally, all the parameters of the equivalent circuit are computed with equations (3)-(6) with the measured quantities. In (4),  $F_{IN}(f_M)$  is expressed in scaled units (by a factor  $m_{eq}$ ) so that it is numerically equal to  $\ddot{y}(f_M)$ . The values obtained for the considered transducers are reported in Table I.

TABLE I. IDENTIFIED MODEL PARAMETERS

mass [g]	$L_M$ [kg]	$C_M$ [m/N]	$R_M$ [N·s/m]	$\alpha$ [N/V]	$\beta$ [A·s/m]	$C_P$ [nF]
6	$5.82 \cdot 10^{-2}$	$2.44 \cdot 10^{-4}$	1.16	$12.2 \cdot 10^{-3}$	$2.10 \cdot 10^{-3}$	52
10	$5.32 \cdot 10^{-2}$	$4.37 \cdot 10^{-4}$	0.96	$4.32 \cdot 10^{-3}$	$3.40 \cdot 10^{-3}$	52
12	$5.85 \cdot 10^{-2}$	$4.80 \cdot 10^{-4}$	1.06	$3.78 \cdot 10^{-3}$	$2.90 \cdot 10^{-3}$	52
18	$7.54 \cdot 10^{-2}$	$5.27 \cdot 10^{-4}$	1.26	$4.90 \cdot 10^{-3}$	$2.10 \cdot 10^{-3}$	52

In order to prove the effectiveness and the degree of reliability of the proposed approach, after identifying their model parameters, Spice simulations of the four previously identified piezoelectric transducers have been compared to direct measurements in different operating conditions.

Fig. 5 shows a comparison between the predicted and the observed behavior of the four transducers in frequency domain. The identified models accurately predict the response of the piezoelectric transducers to vibrations.

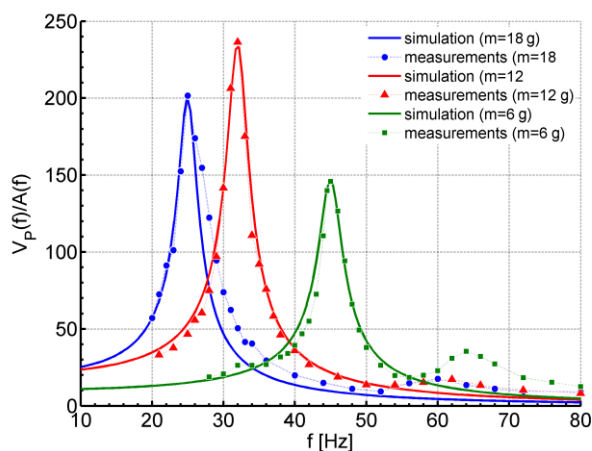


Figure 5. Comparison in frequency domain between measured (dots) and simulated data (continuous lines) for the Q220-A3-303YB transducer with different tip masses of 6g, 12g, 18g in open circuit configuration.  $V_p(f)$  is the output voltage of the transducer,  $A(f)$  is the input acceleration ( $\ddot{y}$ ).

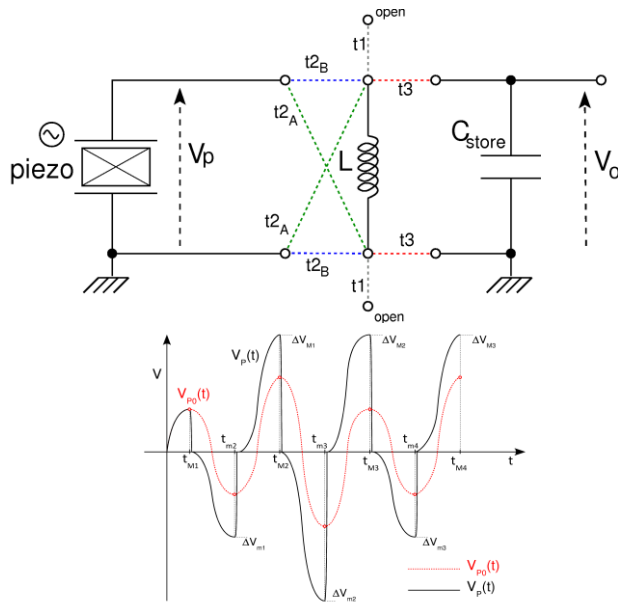


Figure 6. Schematic of the power converter performing synchronous charge extraction (top). Qualitative representation of waveforms when the converter is active and inactive (bottom). Synchronous conversion introduces at each activation a voltage offset, which is expected to increase the peak-to-peak voltage on the transducer.

As an additional proof, another measurement and simulation campaign was performed for verifying the capability of the model of predicting the effects of a non linear power converter on the performance of the transducer. In this setup, the piezoelectric transducers were connected to a switching converter circuit performing synchronous charge extraction. A schematic representation of the power converter is shown in Fig. 6.

Each time that the transducer reaches a local maximum or minimum of voltage it is connected to an inductor for a short time interval until his electric charge (and, consequently, its electrostatic energy) is fully removed. This technique was described in [1] and [5] where weakly coupled transducers with simplified capacitive models (Fig. 1b) were considered. These models predict, in case of sinusoidal vibrations, that the peak-to-peak voltage across the transducer doubles. However, with respect to those analyses, in a general case a lower energy harvesting performance should be expected: in fact, in case of sinusoidal vibrations, a synchronized converter actually applies a periodic series of current pulses to the electrical port of the piezoelectric transducer for removing charge. At resonance this effect cannot be neglected and the overall result is a force opposing to input vibrations. As a consequence, the actual output voltage of the transducer is lower than predicted with a capacitive model with weak electro-mechanical coupling.

A series of time domain simulations was performed on the four transducers and compared to direct measurements: sinusoidal vibrations at different frequencies were generated and the amplitude  $V_{p0}$  of the open circuit voltage was

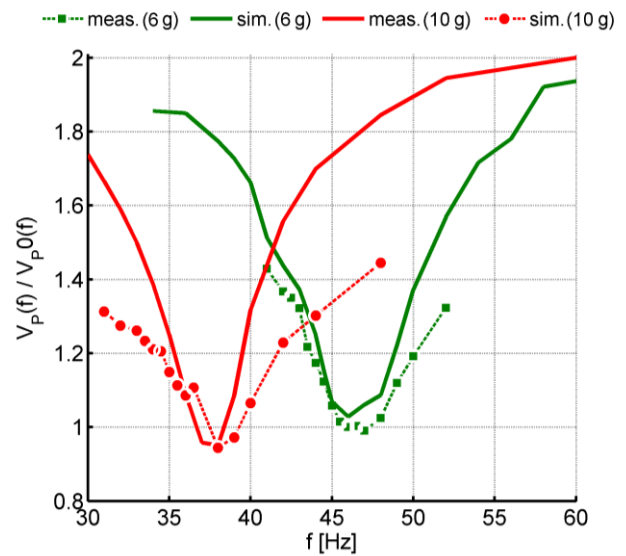


Figure 7. Sinusoidal accelerations of known frequency and amplitude were applied to the Q220-A4-303YB transducer with tip masses of 6g and 10g. The voltage on the terminals of the transducer was measured in an open circuit configuration ( $V_{p0}$ ) and when the power converter performing synchronous charge extraction is connected ( $V_p$ ). The plot compares the measured ratios of the two voltage with values obtained from simulations.

compared to the actual amplitude  $V_p$  observed when the synchronized converter is activated. Fig. 7 shows the  $V_p/V_{p0}$  ratio as a function of vibration frequency. Measurements show that near the resonance the power converter produces a significant and undesired damping effect. However, once again circuit simulations with the identified parameters accurately predict the effective behavior and allow taking it into account in the design phase. On the contrary, the hypothesis of weak electromechanical coupling (i.e. a transducer model such as that in Fig. 1b) would predict a constant ratio  $V_p/V_{p0} = 2$  [1] and thus lead to a significant overestimate of harvester performance.

An accurate identification of model parameters is extremely useful in energy harvesting applications. The available accelerations can be easily measured with cost effective accelerometers and provided as input to the equivalent electro-mechanical circuits of piezoelectric transducers. At the same time, actual power conversion circuits can be jointly simulated. Different topologies or control algorithms of the power converter, such as for example synchronous charge extraction, can be easily characterized. Moreover, their impact on the actual performance of the transducer is determined. The proposed modeling technique allows to choose the most suitable power converter for maximizing the output power depending on the effectively available accelerations.

## V. CONCLUSION

This paper presented a technique for a rapid and reliable identification of model parameters of piezoelectric transducers, with a special focus on energy harvesting applications. All the necessary measurements are carried out with common laboratory instrumentation and cheap equipment. Parameter identification allows performing reliable joint simulations of transducer and electronic circuits. This is of great help during the design phase of piezoelectric energy harvesting systems since it allows better performance estimates even during early design phases.

As a proof of concept it was shown that this technique provides reliable results both in frequency domain and in a transient analysis with a non-linear power converter. In the latter case the proposed model was able to predict undesired side effects at resonance and to overcome the limitations imposed by simplified assumptions or limitative hypotheses commonly taken in literature.

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