Allen's Interval Algebra and Smart-type Environments

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***Abstract*—Allen's interval algebra is a calculus for temporal reasoning that was introduced in 1983. Reasoning with qualitative time in Allen's full interval algebra is nondeterministic polynomial time (NP) complete. Research since 1995 identified maximal tractable subclasses of this algebra via exhaustive computer search and also other *ad-hoc* methods. In 2003, the full classification of complexity for satisfiability problems over constraints in Allen's interval algebra was established algebraically. Recent research proposed scheduling based on the Fishburn-Shepp correlation inequality for posets. This article first reviews Allen's calculus and surrounding computational issues in temporal reasoning. We then go on to describe three potential temporal-related application areas as candidates for scheduling using the Fishburn-Shepp inequality. We also illustrate through concrete examples, and conclude the importance of Fishburn-Shepp inequality for the suggested application areas that are the development of smart homes, intelligent conversational agents and in physiology with emphasis during time-trial physical exercise. The Fishburn-Shepp inequality will enable the development of smart type devices, which will in turn help us to have a better standard of living.**

Keywords*—*Allen's interval algebra; artificial intelligence; qualitative temporal reasoning; scheduling; smart-type reasoning.

I. Introduction

The study of problems involving temporal information or constraints can yield ornate patterns and structures [1]. Temporal reasoning is a mature research endeavor and arises naturally in numerous diverse applications of artificial intelligence, such as: planning and scheduling [2], natural language processing [3], diagnostic expert systems [4], behavioural psychology [5], circuit design [6], software tools for comprehending the state of patients in intensive care units from their temporal information [7], business intelligence [8], and timegraphs, that is graphs partitioned into a set of chains supporting search, which originated in the context of story comprehension [9].

Allen [10] introduced an algebra of binary relations on intervals (of time), for representing and reasoning about time. These binary relations, for example *before*, *during*, *meets*, describe *qualitative* temporal information, which we will be concerned with here. The problem of satisfiability for a set of interval variables with specified relations between them is that of deciding whether there exists an assignment of intervals on the real line for the interval variables, such that all of the specified relations between the intervals are satisfied. When the temporal constraints are chosen from the full form of Allen's algebra, this formulation of satisfiability problem is known to be NP-complete. However, reasoning restricted to certain fragments of Allen's algebra is generally equivalent to related well-known problems such as the interval graph and interval order recognition problems [11], which in turn find application in molecular biology [12][13][14].

The scope of this paper is to explore applications of Allen's interval algebra in the context of smart environments. Further, we combine theory from correlation inequalities with temporal reasoning concepts targeted towards smart-type scheduling applications. The inequality of interest is the Fishburn-Shepp inequality.

This paper is structured as follows: Section II, we describe various applications in temporal reasoning that include applications in smart homes, applications in intelligent conversational agents, and also applications in exercise physiology, followed by Section III, which describes conclusions and future work.

A. Allen's Interval Algebra

Allen's [10] calculus for reasoning about time is based on the concept of *time intervals* together with *binary relations* on them. In this approach, time is considered to be an infinite dense ordered set, such as the rationals $\mathbf{R}$, and a *time interval* X is an ordered pair of time points (X^-, X^+) such that $X^- < X^+$.

Given two time intervals, their relative positions can be described by exactly one of the members of the set $\mathbf{B}$ of 13 basic interval relations, which are depicted in Table I; note that the relations $X^- < X^+$ and $Y^- < Y^+$ are always valid, hence omitted from the table. These basic relations describe relations between *definite* intervals of time. On the other hand, *indefinite* intervals, whose exact relation may be uncertain, are described by a set of all the basic relations that may apply.

The universe of Allen's interval algebra consists of all the binary relations on time intervals, which can be expressed as disjunctions of the basic interval relations. These disjunctions are written as sets of basic relations, leading to a total of $2^{13} = 8192$ binary relations, including the *null relation* $\emptyset$ (also

TABLE I. THE SET $\mathbf{B}$ OF THE THIRTEEN BASIC QUALITATIVE RELATIONS DEFINED BY ALLEN [15].

Basic Interval Relation	Symbol	Endpoint Relations
X precedes (before) Y	p ($\prec$)	$X^+ < Y^-$
Y preceded-by (after) X	$p^\smile$ ($\succ$)	
X meets Y	m	$X^+ = Y^-$
Y met-by X	$m^\smile$	
X overlaps Y	o	$X^- < Y^- < X^+ < Y^+$
Y overlapped-by X	$o^\smile$	
X during Y	d	$X^- > Y^-,\ X^+ < Y^+$
Y includes X	$d^\smile$	
X starts Y	s	$X^- = Y^-,\ X^+ < Y^+$
Y started-by X	$s^\smile$	
X finishes Y	f	$X^- > Y^-,\ X^+ = Y^+$
Y finished-by X	$f^\smile$	
X equals Y	$\equiv$	$X^- = Y^-,\ X^+ = Y^+$

denoted by $\perp$) and the *universal relation* $\mathbf{B}$ (also denoted by $\top$). The set of all binary relations $2^{\mathbf{B}}$ is denoted by $\mathcal{A}$; every temporal relation in $\mathcal{A}$ can be defined by a conjunction of disjunctions of endpoint relations of the form $X < Y, X = Y$ and their negations.

The operations on the relations defined in Allen's algebra are: unary *converse* (denoted by $^\smile$), binary *intersection* (denoted by $\cap$) and binary *composition* (denoted by $\circ$), which are defined as follows:

$$\begin{aligned} \forall\, X,Y: &\quad X r^\smile Y \ \leftrightarrow\ YrX \\ \forall\, X,Y: &\quad X(r \bigcap s)Y \ \leftrightarrow\ XrY \ \bigwedge\ XsY \\ \forall\, X,Y: &\quad X(r \circ s)Y \ \leftrightarrow\ \exists Z : (XrZ \ \bigwedge\ ZsY), \end{aligned}$$

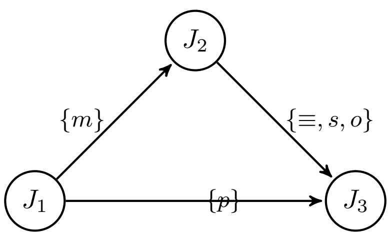
where X, Y, Z are intervals, and r, s are interval relations. Allen [10] gives a composition table for the basic relations.

Fundamental *reasoning problems* in Allen's framework have been studied by a number of authors, including Golumbic and Shamir [16][17], Ladkin and Maddux [18], van Beek [19] and Vilain and Kautz [20].

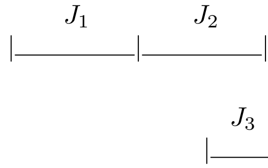
A *graph* is an ordered pair $G = (V, E)$ comprising a set V of *vertices* together with a set E of *edges*, which are 2-element subsets of V; if these subsets comprise ordered pairs of vertices then the graph is said to be *directed*. The following example illustrates a directed constraint graph expressing indefinite qualitative temporal information.

Example 1.1:

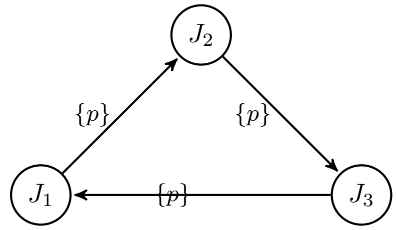
Consider the following CSP (constraint satisfaction problem) where the constraints are the relations of Allen's interval algebra, and each J_i, say job to be scheduled, is a time interval of the form (X^-, X^+).



This temporal constraint satisfaction problem has a unique solution as follows:



In contrast, the following CSP has no solution.



B. Subclasses of Allen's Interval Algebra

In this section we consider restricted temporal reasoning problems in which the relations are chosen from specified subsets of the set of all temporal relations on intervals, $\mathcal{A}$. Note that there are $2^{|\mathcal{A}|}$ such subsets, that is 2^{8192}, or approximately 10^{2466} – clearly a massive combinatorial issue.

For every subset $\Gamma \subseteq \mathcal{A}$ of temporal relations, the corresponding restricted satisfiability problem ISAT(Γ) is equivalent to CSP(Γ) - hence the complexity of ISAT(Γ) can be obtained via the complexity of CSP(Γ).

We now consider some well-known tractable subclasses of Allen's algebra.

Example 1.2 (The continuous endpoint class, $\mathcal{C}$):

This class includes all temporal relations, which may be defined using conjunctions of clauses of endpoint relations of the form $x = y$, $x \leq y$ and $x \neq y$, such that (1) there are only unit clauses, and (2) for each unit clause $x \neq y$, the clause form also contains a unit clause of the form $x \leq y$ or $y \leq x$.

It contains 83 relations, including $\{d,\ o,\ s\}$, $\{s^\smile,\ o^\smile,\ \equiv,\ f\}$, and $\{d^\smile,\ f^\smile,\ o,\ m,\ p\}$, (as well as the null relation, $\perp$). For example, the relation $\{d,\ o,\ s\}$ is defined by the following conjunction of endpoint relations (see Table I):

$$\{(X^- \leq X^+),\ (X^- \neq X^+),\ (Y^- \leq Y^+),\ (Y^- \neq Y^+),\ (X^- \leq Y^+),\ (X^- \neq Y^+),\ (Y^- \leq X^+),\ (X^+ \neq Y^-),\ (X^+ \leq Y^+),\ (X^+ \neq Y^+)\}.$$

The continuous endpoint class was first described and shown to be tractable by Vilain, Kautz and van Beek [21], and subsequently described by Ligozat in terms of "convex relations" with respect to a lattice representation [22]. This subclass has the computational advantage that the path-consistency method solves ISI($\mathcal{C}$) [23], [24], [21].

Example 1.3 (The pointisable class, $\mathcal{P}$):

This slight generalization of class $\mathcal{C}$ is defined in the same way as $\mathcal{C}$, but without the condition (2). It contains 188 relations, including all relations in $\mathcal{C}$ together with (non-convex) relations such as $\{d,\ o\}$, $\{d^\smile,\ o^\smile,\ f^\smile,\ f\}$, and $\{d^\smile,\ f^\smile,\ o,\ p\}$.

This class of temporal relations was first described and shown to be tractable by Ladkin and Maddux [18] and studied by van Beek and Cohen [24]. Although path-consistency is not sufficient for solving ISI($\mathcal{P}$) [23], it is for deciding ISAT($\mathcal{P}$) [18], [20]. Van Beek [23], [25] and van Beek and Cohen [24] give algorithms for solving ISI($\mathcal{P}$) in $O(n^4)$ time; van Beek specifies an algorithm for deciding ISAT($\mathcal{P}$) in $O(n^2)$ time [25].

Example 1.4 (The ORD-Horn class, $\mathcal{H}$):

This class is a strict superset of $\mathcal{P}$, defined using conjunctions of *disjunctions* of the endpoint relations in $\mathcal{P}$, and where each disjunction contains at most one relation, which is not of the form $x \neq y$. That is, the relations permit an ORD-clause form containing only clauses with at most one positive literal. It contains 868 relations, including all those in $\mathcal{P}$ together with relations such as $\{f^{\smile},\ s,\ o\}$, whose endpoint relations are given by the set:

$$\{(X^- \leq X^+), (X^- \neq X^+), (Y^- \leq Y^+), (Y^- \neq Y^+), (X^- \leq Y^-), (X^- \leq Y^+), (X^- \neq Y^+), (Y^- \leq X^+), (X^+ \neq Y^-), (X^+ \leq Y^+), (X^- \neq Y^- \ \vee \ X^+ \neq Y^+)\}.$$

Nebel and Bürckert [15] identified this, via machine enumeration, to be the first known maximal tractable subclass, and, the unique greatest tractable subclass amongst those that contain all 13 basic relations – comprising over 10% of the full algebra. Further, they established that the path-consistency method is sufficient for deciding ISAT($\mathcal{H}$), implying its wider applicability [15].

Ligozat [26] showed that any subalgebra which contains all basic relations, and a relation, which is not ORD-Horn, will contain at least two of four "corner" relations: $\{d^{\smile},\ s^{\smile},\ o^{\smile},\ f,\ d\}$, $\{o^{\smile},\ s^{\smile},\ d^{\smile},\ f^{\smile},\ o\}$ and their converses.

Example 1.5 (Starting point and ending point algebras):

Drakengren and Jonsson [27] discovered a large family of maximal tractable subclasses, "starting point" and "ending point" algebras, denoted $S(b), S^*,\ E(b), E^*$ - the parameter b is chosen from specified basic relations. The six algebras $S(b)$ & $E(b)$ contain 2312 elements each, and $S^\star$ & $E^\star$ contain 1445 elements each. For brevity we only define $S(b)$; for $E(b)$, $S^\star$ and $E^\star$ see [27].

Let $r_s = \{\succ,\ d,\ o^{\smile},\ m^{\smile},\ f\}$, and let $r_e = \{\prec,\ d,\ o,\ m,\ s\}$. Then, for $b \in \{\succ,\ d,\ o^{\smile}\}$, define $S(b)$ to be the set of relations r such that:

$$\begin{aligned} \{b, b^{\smile}\} &\subseteq r \\ \{b\} \subseteq r &\subseteq r_s \cup \{\equiv,\ s,\ s^{\smile}\} \\ \{b^{\smile}\} \subseteq r &\subseteq r_s^{\smile} \cup \{\equiv,\ s,\ s^{\smile}\} \\ r &\subseteq \{\equiv,\ s,\ s^{\smile}\}. \end{aligned}$$

The algebras allow for metric constraints on interval starting or ending points.

Initially, various computer-assisted exhaustive searches led to a classification of complexity within parts of Allen's algebra [27], [28], [29]. For further progress, it was understood that theoretical studies of the structure of Allen's algebra would be necessary, since using these methods to obtain a classification would require dealing with more than 10^{50} individual cases - clearly not feasible.

We proceed to consider an algebraic approach to the characterisation of tractable subclasses of relations.

C. Algebraic Closure Properties of Constraints

Jeavons *et al.* [30] developed a theory of *algebraic closure properties of constraint relations*, which can be used to distinguish between sets of relations, which give rise to tractable CSPs and those which give rise to NP-complete CSPs. In this theory the significant algebraic properties of a relation are the operations under which it is *closed*: in the sense of the following definition.

Definition 1.1: [30] Let R be an n-ary relation over a domain D, and let $\varphi : D^k \rightarrow D$ be a k-ary operation on D. The relation R is said to be *closed under* φ if, for all $t_1, t_2, \ldots, t_k \in R$, $\varphi(t_1, t_2, \ldots, t_k) \in R$, where $\varphi(t_1, t_2, \ldots, t_k) =$ $\langle\varphi(t_1[1], t_2[1],\ \ldots, t_k[1]),\ \varphi(t_1[2], t_2[2], \ldots, t_k[2]),\ \ldots,$ $\varphi(t_1[n], t_2[n], \ldots, t_k[n])\rangle$.

The algebraic approach to tractability refers to special properties of operations including: *idempotent, constant, unary, projection, semiprojection, majority, affine, or ACI (associative, commutative & idempotent)* - details in [30].

Further theoretical advances followed. In 2003, Krokhin *et al.* [31] completed the analysis of complexity for satisfiability problems expressed in Allen's algebra by showing that all known maximal subclasses were the only forms of tractability within this interval algebra: Allen's algebra contains exactly eighteen maximal tractable subalgebras and that reasoning within any subset not included in one of these is NP-complete.

Their purely analytical method makes extensive use of the operations defined in the algebra (converse, intersection and composition), while exploiting the fact that tractability of a subalgebra is a pertinent hereditary property in Allen's algebra. Importantly, both the result and the algebraic method can be used to classify the complexity in other temporal and spatial formalisms.

See [32] for a fuller account of this survey of temporal reasoning. The discussion of complexity and related computational issues leads naturally to the next section involving heuristics.

D. Posets and the Fishburn-Shepp Inequality

We now consider novel research proposed in [32], namely, to specify heuristics for scheduling based on representing a collection of intervals of time with constraints as a poset, and applying the Fishburn-Shepp inequality to guide a scheduling algorithm. In [32], applications are sought for this approach: we address this first step here by describing potential applications, which are also related to smart-type reasoning. First, we commence with overviews of the scheduling problem and the Fishburn-Shepp inequality.

Generally, a *schedule* of tasks (or simply schedule) is the assignment of tasks to specific time intervals of resources, such that no two tasks occupy any resource simultaneously – additionally, a requirement can be that the capacity of resources is not exceeded by the tasks. A schedule is *optimal* if it minimizes a given optimality criterion. However, our ultimate interest is in providing an algorithm to solve, or schedule, temporal constraint satisfaction problems; since we also consider indefinite qualitative temporal information, the solution may assign events simultaneously to intervals.

Let Q be a finite *poset* (partially ordered set) with n elements and C be a chain $1 < 2 < \cdots < c$. For (Q, C), a map $\omega : Q \to C$ is *strict order-preserving* if, for all $x, y \in Q$, $x < y$ implies $\omega(x) < \omega(y)$. Let $\lambda : Q \to \{1 < 2 < \cdots < n\}$ be a *linear extension* of Q, that is, an order-preserving injection.

A poset Q is equivalently a *directed acyclic graph (DAG)*, $G = (V, E)$; for temporal reasoning, the vertices represent time intervals, and edges between vertices are labeled with relations in Allen's algebra, which satisfy the partial ordering. For scheduling problems, a linear extension λ of Q (or G) can be used to schedule tasks: λ must respect interval constraints, that is relations between comparable elements. Algorithmically, a linear extension of a DAG, G, can be determined in linear time by performing a depth-first search of G; while G (Q) can be represented by an adjacency matrix.

The Fishburn-Shepp inequality [33] [34] is an inequality for the number of extensions of partial orders to linear orders, expressed as follows. Suppose that x, y and z are incomparable elements of a finite poset, then

$$P(x < y)P(x < z) < P((x < y) \wedge (x < z)) \qquad (1)$$

where $P(*)$ is the probability that a linear extension has the property *. By re-expressing this in terms of conditional probability, $P(x < z) < P((x < z) \mid (x < y))$, we see that $P(x < z)$ strictly increases by adding the condition $x < y$. The problem posed in [32] concerns applying the Fishburn-Shepp inequality to efficiently find a favourable schedule under specified criteria, where a naive scheduling algorithm is also given together with an illustrative example. However, our focus here is in introducing application scenarios.

II. Applications in Temporal Reasoning

A. Applications in Smart Homes

Buildings consume a considerable amount of energy. Managing that energy is challenging, though is achievable through building control and energy management systems. These systems will typically monitor, measure, manage and control services for the lighting, Heating, Ventilation and Air Conditioning (HVAC), security, and safety of the building. They also permit a degree of scheduling, albeit they are often limited by static programming and may have no awareness incorporated of external events. For example, a building's HVAC system

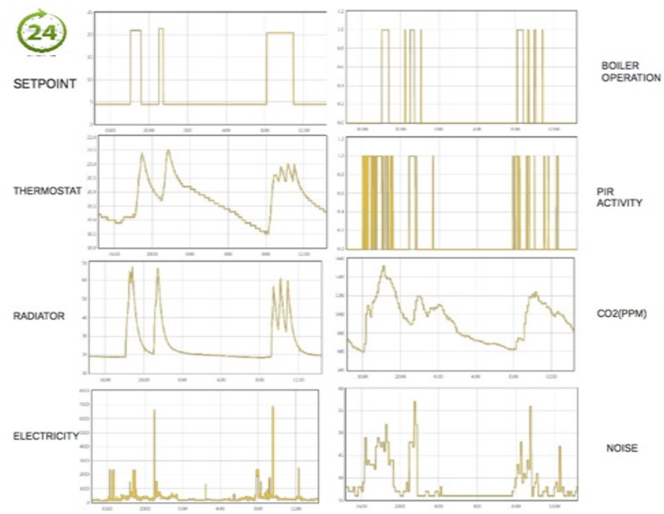


Fig. 1. Smart Home Temporal Data captured over 24 hours.

may heat rooms that are unoccupied as the setpoint has been programmed to be a certain temperature for a specified interval of the day (Figure 1). Clearly this is quite inefficient, and though motion detectors can play a role in actuating lights during periods of room occupancy, maintaining a comfortable indoor climate using similar sensors to detect people cannot provide the same benefits. Furthermore, the indoor climate is impacted by outdoor thermodynamic processes, as well as internal heat gains, which can be unaccountable (e.g., people, mobile equipment, etc). However, most modern non-residential building's energy management systems will be configured to turn building services on and off throughout the day using a pre-programmed schedule (e.g., a repeating daily pattern of heating use) and can also employ intelligent start-up controllers with external temperature compensation to delay the turning on of heating for example. Modern heating controllers (i.e., programmable thermostats) in homes can also have setpoints configured in a daily schedule (e.g., 6-8am: increase setpoint to 20°C, representing a waking-up phase; 9am - 4pm: heating deactivated or set to a maximum (e.g., 15°C); and from 5pm - 6pm: 21°C, representing a heating-up phase to anticipate arrival of an occupant from a workplace, and so forth).

Aside from heating control, homes can now also employ smart home systems to perform some degree of energy management and appliance automation. These systems are becoming more commonplace, particularly as the Internet of Things (IoT) paradigm is gaining more traction, whereby humans are bypassed, and machine to machine communication takes place (e.g., Smart Homes communicating with Smart Grids [35]) [36]. This gives rise to smart automation and reasoning where decision making can take place and determine when home appliances can be scheduled, particularly in the case of peak-load shaving [37] or demand response optimization [38]. In these cases, consumption patterns can be shifted to times of lower cost electricity. Appliance scheduling can be further classified by, for instance, their minimum required periods of operations, whether or not their operations can be interrupted, and if a human occupant is involved (i.e., in climate control scenarios). For instance, washing machines will have varying periods of operation depending on the program (wash, spin, dry) and cannot (typically) be interrupted if scheduled. Heating or cooling systems will have optimum start-up times to turn on in anticipation of occupants requiring the temperature of the house to be at a preset setpoint upon arrival. The Internet of Things has even enabled this particular scenario to be influenced by the distance an occupant is from the home or building, whereby the driving time is estimated via tracking of a Global Positioning System (GPS signal) [39]. In [40], driving patterns were analysed, and a programmable thermostat augmented with GPS control enabled energy savings of 7%.

The emerging Internet of Things in this respect will be responsible for huge volumes of temporal pattern data as shown in Figure 1 (i.e., timestamped sequences of events, be it a change in temperature, or a light being turned on and off, or the duration of activity of an entertainment system, etc [41]), thus also incorporating quantitative temporal information. In the smart home, the ability to detect user behaviour or house activities from this kind of temporal pattern data can provide a better understanding of how to identify patterns of energy use and thus determine when or how to gain energy savings. Naturally, the accumulative savings factor is increased many-fold in the smart city concept. Temporal pattern event detection inspired by Allen's relations has proved useful in smart environments: for anomaly detection in assisted living applications [42], and in activity monitoring [43]. In these examples, intervals represent the sensed data (cooking would imply the stove being on while an inhabitant is present in the kitchen [44] [45]). Such kinds of recognition are useful for determining normal behaviour of elderly occupants, and thus, for instance, detecting any onsets of dementia [46].

Clearly, efficient, or ideally optimized, scheduling of events can lead to enhanced savings of time and energy – it is with this goal that we propose applying the Fishburn-Shepp inequality, possibly to a specified subset of events in a larger complex system. Regarding this goal we consider Example 8 from [32] and illustrate applying it to a potential smart-type environment.

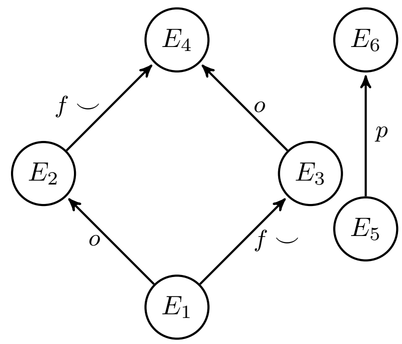
Example 2.1 (adapted from Example 8 in [32]): Consider a set of time intervals of events (alternatively jobs as in [32]), $\mathcal{E} = \{E_1, E_2, E_3, E_4, E_5, E_6\}$ with a partial order $\leq$ defined by $(X^- < Y^-) \wedge (X^+ \leq Y^+)$, which requires scheduling – note the partial order defines a subclass of Allen's interval algebra. Suppose that $\mathcal{E}$ is the poset / DAG shown below, where the edges have been labeled with satisfying temporal relations, and there are two triples of incomparable elements: $\{E_2, E_3, E_5\}$ and $\{E_2, E_3, E_6\}$. Then

$P(E_2 < E_3) \wedge (E_2 < E_5)) = 6/30$, $P((E_3 < E_2) \wedge (E_3 < E_5)) = 6/30$, and $P((E_5 < E_2) \wedge (E_5 < E_3)) = 18/30$;

$P((E_2 < E_3) \wedge (E_2 < E_6)) = 12/30$, $P((E_3 < E_2) \wedge (E_3 < E_6)) = 12/30$, and $P((E_6 < E_2) \wedge (E_6 < E_3)) = 6/30$.

The largest set of linear extensions corresponding to the first triple is when $(E_5 < E_2) \wedge (E_5 < E_3)$ and for the second triple is (w.l.o.g.) when $(E_2 < E_3) \wedge (E_2 < E_6)$. Their intersection has 6 linear extensions and we arbitrarily choose $E_1 < E_5 < E_2 < E_6 < E_3 < E_4$ as a schedule – so, although any linear extension would suffice we are selecting one which appears more "suitable" to satisfy: this illustrates the heuristic – the proposed algorithm is described in [32].

A parallel solution to this temporal constraint satisfaction problem from [32] is: $E_1 = (10, 20)$, $E_5 = (5, 10)$, $E_2 = (12, 23)$, $E_6 = (20, 22)$, $E_3 = (15, 20)$ and $E_4 = (18, 23)$. We will proceed to make this somewhat abstract schedule more concrete in a smart scenario.



Suppose that the time intervals corresponding to the duration of events are interpreted in an integrated smart working and home environment as follows.

- E_1 denotes working in a flexible mode such as driving between locations and using mobile devices.
- E_2 denotes commuting home.
- E_3 denotes smart HVAC-type home ventilation.
- E_4 denotes activating and running a home appliance such as an oven.
- E_5 denotes robotic mowing of the lawn.
- E_6 denotes automated watering of the mowed lawn.

The temporal relations for this example are clarified as follows:

- E_1 *overlaps* E_2 as the employee could optimize work flow to organize driving home while completing their work assignment.
- E_2 *finishes* $\smile$ E_4 (E_2 *finished-by* E_4) as the GPS system tracking the employee's return home will synchronize with the oven timer so that the meal is cooked when the employee arrives.
- E_1 *finishes* $\smile$ E_3 (E_1 *finished-by* E_3) as the GPS tracker will also synchronize with the ventilation system so that the house is aired appropriately.
- E_3 *overlaps* E_4 as the ventilation should be completed before the meal is ready as the HVAC heating or air conditioning is likely to be activated with the residents' meal.
- E_5 *precedes* E_6 as the [electric & electronic] robot must return to a storage area before the watering starts.

Then an alternative "suitable" schedule (from the set of 6 linear extensions) can be specified as: $E_1 < E_5 < E_2 < E_3 < E_4 < E_6$. A solution using a 24-hour clock is given by: $E_1 = (09:00, 17:00)$, $E_5 = (10:00, 11:00)$, $E_2 = (16:15, 17:30)$, $E_3 = (15:30, 17:00)$, $E_4 = (16:30, 17:30)$ and $E_6 = (23:00, 24:00)$. Note that the noisy lawn mowing occurs after the resident has gone to work, while the garden is watered during an off-peak water consumption period and also when there is no sun.

We demonstrate another abstract type example of a "suitable" schedule for this poset, again with $(E_5 < E_2) \wedge (E_5 < E_3)$, but this time choosing $(E_3 < E_2) \wedge (E_3 < E_6)$. Then a linear extension is $E_5 < E_1 < E_3 < E_6 < E_2 < E_4$. A solution is $E_5 = (5, 10), E_1 = (10, 20), E_3 = (14, 20), E_6 = (15, 25), E_2 = (16, 50), E_4 = (18, 50)$. Note that each edge on this chain satisfies the partial order $\leq$. We conclude this section with:

Question, does this heuristic based on the Fishburn-Shepp inequality make finding a solution to a temporal constraint satisfaction problem easier?

B. Applications in Intelligent Conversational Agents

Intelligent conversational agents (CA) enable natural language interaction with their human participant. Following an input string, the CA works through its knowledge-base in search of an appropriate output string. The knowledge-base consists of natural language sentences based on a specific domain. Through the use of semantic processing using a lexical database with grouped sets of cognitive synonyms, word similarity is determined, with thus the highest semantically similar ranked string returned to the user as output.

Scripts consist of contexts that relate to a specific theme or topic of conversation. Each context contains one or more rules, which possess a number of prototype natural language sentences. An example of a scripted natural language rule is shown below, where s is a natural language sentence and r is a response statement.

<Rule-01>

s: I am having problems accessing my email account.
r: I'm sorry to hear that. Have you tried contacting IT support?

One such CA, as proposed by O'Shea *et al.* ([47] [48] [49]), uses semantics as a means to measure sentence similarity. The semantic-based CA is organized into contexts consisting of a number of similarly related rules. Through the use of a sentence similarity measure, a match is determined between the user's utterance and the scripted natural language sentences. Similarity ratings are measured in the range from 0 to 1, in which a value of 0 denotes no semantic similarity, and 1 denotes an identical sentence pair. The highest ranked sentence is 'fired' and its associated response is sent as output. The following algorithm describes the application:

1. Natural language dialogue is received as input from the user.

2. Semantic-based CA receives natural language dialogue from the user, which is sent to the sentence similarity measure.

3. Semantic-based CA receives natural language sentences from the scripts files, which are sent to the sentence similarity measure.

4. Sentence similarity measure calculates a firing strength for each sentence pair, which is returned and processed by the semantic-based CA.

5. The highest ranked sentence is fired and its associated response is sent as output.

The use of CAs can be applied to educational settings to better support teaching and learning and provide alternative learning environments. Computing underpins almost all areas of the modern world and many new opportunities in science and engineering could not have been realized without it. Thus, computer scientists have the potential to revolutionize societies, develop economic wealth and change peoples' lives. Employers need to ensure that the future generations have the skills required, which is only achieved through effective teaching and learning practices at our schools. If we do not engage at this level, we are unlikely to produce the number of computer scientists required to satisfy the demands of industry. However, teachers need support from the science community and industry to increase the number of students (especially girls) leaving school confident in coding, algorithmic thinking and computer science.

In order to develop knowledge exchanges, evidence is needed to assess its provision and accessibility. This would lead to studies discussing effective pedagogy and research into developing alternative approaches for engagement. However, opportunities may exist using alternative approaches, including artificial intelligence (AI). Artificial intelligence within teaching and learning is an opportunity to seek novel ways to deliver curriculum content. This may come in the form of a CA using intelligent adaptive learning methodologies. Such CAs or learning companions can accompany students asking questions, providing encouragement, offering suggestions and connections to resources, and help talk through difficulties as a teacher would demonstrate in class. Over time, the companion would 'learn' what you know, what interests you, and what kind of learner you are. Such research would raise ethical questions to the use of AI applications alongside human educators; however is it a challenge that will impact profoundly on the nature of teaching and learning? There is enormous potential for such research that remains undiscovered along with its associated benefits. AI will empower teachers, which will better inform them on how to manage the learning of their students.

The use of CAs using intelligent learning methodologies may also provide a means to support different types of conversations and thus, learning styles. The premise for such CAs is to guide students through the questioning process as opposed to simply providing an answer. Students will gain independence by answering questions for themselves through appropriate dialogue. This appropriate dialogue can be modelled using Allens interval algebra: the intervals of speech could satisfy the basic relation p, if one speaks before the other, or the relation o if their speech overlaps, and so on. In terms of scheduling a set of speech events with specified relations, that is constructing a linear extension by applying the Fishburn-Shepp inequality, we envisage an application for the learning impaired, which schedules the events sequentially to reduce confusion from simultaneous speech. This could then be integrated with the CA technology.

C. Applications in Physiology

In exercise physiology, the study of complex rhythms arising from the peripheral systems (for example, the cardiovascular system) and the central nervous system of the human body is important to optimize athletic performance while using a suitable type of pacing. Pacing plays an important part during athletic competition so that the metabolic resources are used effectively to complete the physical activity in the minimum time possible, as well as to maintain enough metabolic resources to complete that task [50].

Moreover, according to the Central Governor Model (CGM) [51], there is a central regulator that paces the peripheral systems during physical activity to reach the endpoint of that physical activity without physiological system failure. This central governor model of fatigue is a complex integrative control model, which involves the continuous interaction, in a deterministic way, among all the physiological systems, and that of the central system. There has been good evidence that the brain decreases its neural drive to protect the body from irreversible damage. The brain subconsciously controls the status of all systems of the body and it continuously calculates the metabolic costs to continue at the current pace as well as it compares that to the existing physical state. Based on this important information the brain adjusts the optimum pace so that the physical task can be completed in the most efficient way while simultaneously maintaining the overall homeostasis of the body by sustaining physical and mental capacity reserves. The brain protects the body through the regulation of power output during any form of physical exercise (especially long distance running or cycling exercise) with the ultimate aim of maintaining the body's homeostasis and protecting life.

Fig. 2 shows the CGM model of exercise regulation [52], which proposes that it is the brain that regulates exercise performance by changing continuously the number of motor units that are recruited in the exercising limbs. This change occurs in response to conscious and subconscious feedbacks that are present before and during the physical activity.

The goal of this central controller is to ensure that one always exercises with reserve and terminates the physical exercise bout without catastrophic failure of the body systems. The brain employs the feelings of fatigue to control the exercise intensity and duration of maintaining that exercise intensity so that these factors are always within the exercisers' physiological capacity. Therefore, Allen's interval algebra basic relations (precedes, meets, overlaps etc.) can be used to express those time intervals of switching among different pacing in a particular endurance physical activity for instance. The switching times between different pacing strategy themselves depend on many physiological factors (that have relation with time of experiencing that factor) including exogenous factors such as surrounding temperature (change in temperature), change in air pressure especially running at higher altitudes and endogenous factors such as one's metabolic energy reserve (that are different at different times), accumulation of lactic acid (low or high depending on the duration of the exercise and the physiological processes' time to discard it), fluid loss through sweating depends on body heat, which changes through time, level of muscle damage as well as level of skeletal muscle fatigue change through time.

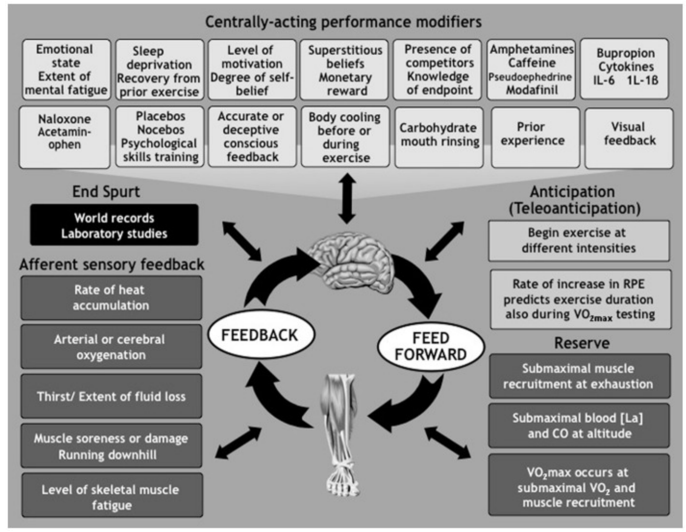


Fig. 2. The Central Governor Model of Exercise Regulation [52]

In this context, the decision making process involved when an athlete changes his or her pacing strategy during a particular race (and especially during endurance exercise) seems quite complex. However, the change in the decision making process could be simply explained by the basic relations in Allen's interval algebra. Consider the following scenario, shown in Fig. 3, where an athlete or runner needs to complete a 20-km race. An experienced runner will subconsciously be aware of the amount of energy resources they will need during the race so that they can effectively complete the race without catastrophic failure. During the race, there are both exogenous and endogenous factors, which will influence the optimal performance of the runner, and therefore she or he has to make important decisions as to when, or when not, to change their pacing during the race so that they can complete the race in the minimum time possible. For instance, there may be three major changes in the patterns of the running speed, power output, or pacing strategies that the runner could adopt for a long distance race such as the 20-km race [53]. Initially, on the first stage of the race, he or she will accelerate from a resting standing (or crouching) position to reach a constant optimal speed as determined by the runner's physical ability; meanwhile their heart rate (HR) will accelerate as well as their volume of oxygen consumption (VO2). In the second stage, they will maintain the same constant running speed for most of the race while their heart rate will be quite steady; moreover, the volume of oxygen consumption will be kept practically

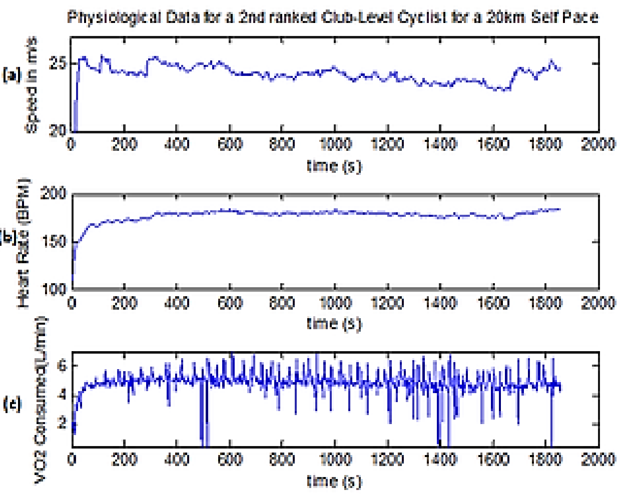


Fig. 3. Physiological data for a 2nd ranked club level cyclist for a 20km self pace under laboratory conditions under ambient body temperature

constant throughout the race. Finally, in the third stage of the race, the runner will accelerate or sprint in order to complete the race, which will at the same time, increase their heart rate as well as the rate of volume of oxygen consumption.

This represents one possible scenario that may occur during a race, which illustrates that Allen's temporal relations can be exploited to more clearly express the complex decision-making processes related to the human body during physical exertion, and hence allow for scheduling the pacing strategy adopted by a runner during a particular race. In fact, the Allen's interval algebra can be applied in smart-type devices, which in turn can help an athlete or sportsman in their decision-making process in real time. For instance, these smart-type devices can integrate various biomarkers computed from measured biosignals or physiological data (such as heart rate, pulse rate, sweating rate, volume of oxygen consumption) of the human body under physical activity (for example, running or cycling). These intelligent devices will help the users or the athletes in making important decisions by providing them useful biofeedbacks and will notify them whether they need to slow down to avoid catastrophic physiological systems failure, or they can just continue with same pace or speed.

III. CONCLUSION AND FUTURE WORK

This paper has brought together our previous findings with support for further and future developments. The management for appropriate dialogue using CAs or learning companions has been highlighted with the potential to support and enhance teaching and learning. This would be applied through the use of novel AI technologies and the application of intelligent conversational approaches using scheduling.

Previous research in temporal pattern reasoning surrounding smart homes has largely focused on activity recognition of inhabitants, and gaining an understanding from sensor data retrieved from indoor environments (such as electricity, temperature, light, or motion). The Internet of Things, however, will provide further dimensions of data from people (wearable sensors, tracking of GPS, etc.). This kind of outdoor data will provide additional context to the smart home and enable it to make better and more informed decisions as to how to actuate and control building services.

For example, returning to the case of augmented heating control using GPS - an occupant leaves the house and goes for a short jog (automatically disabling the heating as they leave) - as they run their own body temperature rises. The wearable sensors will be monitoring their temperature and their GPS coordinates. As they return and approach their home, the augmented heating control with the GPS system will turn on the heating, but will also take into account the occupant's current body temperature, and accordingly apply the appropriate heating control strategy (i.e., reducing the return-to-home setpoint from a previously higher setting and actuation time). In this case, the quantitative temporal information between arrival and heating activation will be lengthened as the temperature setpoint requirement will be reduced. This is just one of a myriad of possibilities that can be realized from the abundance of potential sensor data generated from the Internet of Things. We believe the relation between indoor and outdoor sensing (as well as any other sensing source for that matter) and reasoning strategies requires further exploration, and as part of our future research strategy we will investigate smart home event and action temporal reasoning from multiple data streams beyond enclosed indoor scenarios. In particular, smart-type scheduling is a key factor in energy-related issues.

We envisage enhanced synergy in the smart-environment by integrating intelligent CAs. Useful responses to even simple sentences such as *Where are my keys?* can have impact on human energy and stress levels and allow for more efficient use of time.

To date, physiological research into pacing strategies has focused on the amount of energy resources that are available for a runner to complete a long distance race. Also, pacing is crucial for improving human performance in time-trial physical exercise. Therefore, we propose that the future area in, which the exercise physiology field should endeavour to concentrate more on, is the optimal time in switching between the different types of pacing strategies, so that a race is completed successfully and in the minimum time possible without body systems' homeostatic failure. In order to achieve this, the various changes in pacing, namely, increasing pace, constant pace or decreasing pace, depends on each individual's resource capacity and endurance for each type of pacing so as to achieve the target or complete a physical endurance activity in the least possible time.

Therefore, we suggest that the decision-making process underlying the choice of the various pacing strategies can be informed through the application of Allen's time interval algebra, and the resulting scheduling can be easily controlled and applied to promote and improve world elite athletic performance. For example, the impact of facilitating the decision-making process of an elite athlete is enormous as he or she will feel more comfortable to tap into their inner optimized physical potentials, and hence boost overall confidence, which may contribute further to greater physical performances.

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