

## Mode Selection Algorithm for Advanced TOA Trilateration Techniques

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**Abstract**—The location detection technology (LDT) with various applications is one of the core techniques for the mobile communication system. The time-of-arrival (TOA) trilateration, which is a representative approach of network-based LDTs, estimates the location of a mobile station (MS) using an intersection point of three circles based on signals from at least three base stations (BS). Since the distance between MS and BS is generally estimated by the number of delay samples and it is an integer number, the radius of circles is usually increased and three circles may not meet at a point, which results in the serious estimation error. In order to overcome this problem, the shortest distance and the line intersection algorithms for the general case and the comparison approach of intersection distances for the specific case have been recently proposed. In this paper, we provide the selection methodology between these two cases for using the line intersection algorithm or the comparison approach of intersection distances, for the MS location estimation. The selection procedure for both cases is based on comparing the radiuses of two large circles to distances between four intersection points of a small circle with others and center coordinates of corresponding large circles.

**Keywords**—location detection; time-of-arrival; trilateration; mode selection; three circles intersection.

### I. INTRODUCTION

Recently, the location estimation of MS has received a great attention, because a lot of information related to the MS location is being utilized in many communication services such as location based services (LBS). More and more researches focus on estimating the accurate MS location with the low cost, the high performance, and the reliability of the components. In USA, the MS location estimation should follow the requirement of locating emergency 911 (E-911) services [1]. LBSs involve the ability to find the geographical location of MS and provide services based on its location. The main objective of these services is to assist with the exact information in real time at the right place likes finding patient, children, elder person, transportation services, location of food court, and necessary things [2]-[6].

For estimating the location of MS, several techniques are commonly utilized based on the received signal strength (RSS), TOA, angle of arrival (AOA), and time difference of arrival (TDOA) [7][8]. The TOA trilateration method, which is one of representative location estimation approaches, estimates the MS location using the intersection point of

three circles with their centers corresponding BS coordinates and radiuses corresponding the distances between MS and BSs. However, since we generally estimate the distance between MS and BS counting the number of delay samples, which is an integer, the estimated distance may be slightly increased and three circles based on the radiuses corresponding to the estimated distance may not intersect at a single point. Therefore, three circles have total six intersection points causing estimation error for the accurate location of MS. In order to solve this problem, recently, the shortest distance algorithm [9] and the line intersection algorithm [10] have been proposed for the general case. In general, the line intersection algorithm has better performance than the shortest algorithm, because it considers the increasing factor of the estimated circles. However, its estimation performance may be degraded for the specific case, in which a small circle is located inside the area of two large circles and it intersects two large circles at four intersection points. In order to improve this problem, the comparison approach of intersection distances for the specific case has been proposed in [11]. It calculates four distances between two neighbor intersections among four intersection points of a small circle and two large circles, and compares them. From the compared result, we select the shortest distance and determine the averaged coordinate of two intersection points corresponding to the shortest distance as the estimated location of MS.

Although this approach has better performance than the line intersection algorithm for the general case, it has worse performance than the line intersection algorithm for the above specific case. Thus, we should select better algorithm according to both cases for the optimized performance. In this paper, we propose a mode selection algorithm for using the line intersection algorithm or the comparison approach of intersection distances, according to two cases of the general case or the specific case. In order to select the proper mode between both approaches, the proposed algorithm considers four intersection points related to a small circle and other two large circles. Note that we consider three circles with a small circle and two large circles in the specific case. For the proposed method, we calculate the distance between one of four intersection points and the center of the circle which is not related to the corresponding intersection. We repeat the previous calculation for all four intersections and get four distances from the calculated results. Finally, this algorithm compares each distance to the radius of the circle related to

the center used to calculating its distance. If all four distances are shorter than the corresponding radiuses, we determine that it is the specific case and select the comparison approach of intersection distances for the advanced TOA trilateration technique. However, if at least one distance is longer than them, we determine that it is the general case and select the line intersection algorithm.

The rest of the paper is organized as follow: Section II describes TOA trilateration algorithms, i.e. the line intersection algorithm for the general case and the comparison approach of intersection distances for the specific case. Section III describes the proposed mode selection algorithm for differentiating the general case and the specific case. The performance of the proposed algorithm is discussed by computer simulation results in Section IV. Finally, the conclusion is presented in Section V.

## II. ADVANCED TOA TRILATERATION ALGORITHMS

The trilateration technique estimates the location of MS using the intersection points of three circles based on radiuses corresponding to distances between MS and BSs and center corresponding to coordinates of BSs. The Euclidean distance between MS and the  $i$ th BS is given by

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \quad i = 1, 2, 3 \quad (1)$$

where  $(x, y)$  is the true coordinate of MS and  $(x_i, y_i)$  is the coordinate of the  $i$ th BS. The location of MS is determined by a unique intersection point of three circles, shown in Figure 1. The distance between MS and BS is usually estimated counting the number of time delay samples. Since that number must be an integer, it is given by

$$n_i = \text{ceil}\left(\frac{r_i}{c} \times f\right) \quad (2)$$

where  $n_i$  is the number of delay samples, "ceil" is the round up function used to make an integer value,  $f$  is the particular carrier frequency, and  $c$  is the velocity of the light. The increased distance between MS and BS due to counting the number of delay samples,  $ed_i$ , is given by

$$ed_i = \frac{n_i \times c}{f}. \quad (3)$$

Since the radius of circle may be increased by the increased distance, three circles may not meet at a single point and they have six intersection points, which results in the location estimation error shown in Figure 2. In order to solve this problem, recently, a couple of algorithms have been proposed. In this paper, we consider two advanced TOA trilateration algorithms such as the line intersection algorithm having good performance in the general case and the comparison approach of intersection distances having good performance in the specific case.

### A. Line Intersection Algorithm

Three circles based on the estimated distances between MS and BSs do not intersect at a unique point, because each

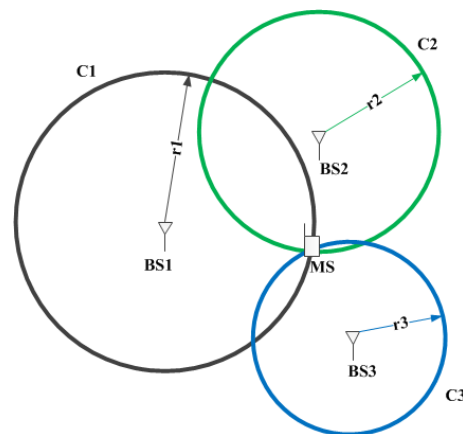


Figure 1. Three intersecting circles to locate MS at a unique point in the general case.

TABLE I. LINE INTERSECTION ALGORITHM

1. Calculate the estimated distance between each BS and MS after counting number of time delay sample, and is given by $ed_i = \frac{n_i \times c}{f}.$
2. Generate three circle equations using estimated distance $\hat{x}^2 + \hat{y}^2 + a_i \hat{x} + b_i \hat{y} + c_i = 0$ where $a_i = -2x_i, b_i = -2y_i$ , and $c_i = x_i^2 + y_i^2 - ed_i^2$ .
3. Obtain the estimated location of MS $(\hat{x}, \hat{y})$ , using $\hat{x} = \frac{(c_2 - c_1)(b_2 - b_3) - (c_3 - c_2)(b_1 - b_2)}{(a_1 - a_2)(b_2 - b_3) - (a_2 - a_3)(b_1 - b_2)},$ and $\hat{y} = \frac{(a_1 - a_2)(c_3 - c_2) - (c_2 - c_1)(a_2 - a_3)}{(a_1 - a_2)(b_2 - b_3) - (a_2 - a_3)(b_1 - b_2)}.$

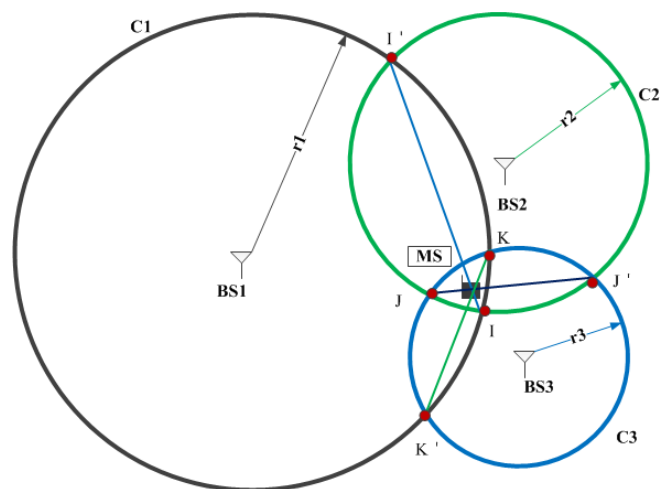


Figure 2. Line intersection at a single point to estimate MS location in the general case.

radius corresponding the estimated distance is generally increased comparing to the original radius. Based on the calculated radiuses, they meet at six intersection points. From the six relevant intersection points, we make three straight lines which connect two intersection points for specific two circles among entire three circles. These three

straight lines must meet at a point and this algorithm determines the intersection point of three lines as the location of MS as shown in Figure 2. Table I summarizes the line intersection algorithm in the general case.

### B. Comparison Approach of Intersection Distances

Although the line intersection algorithm has good performance for estimating the MS location in general case, it may have the poor performance in the specific case, where a small circle is located in the area of two large circles, shown in Figure 3. In order to improve this problem, the comparison approach of intersection distances has been proposed, specialized in the specific case. This algorithm focuses on distances between two neighboring points of four interior intersections related to a small circle, among six entire intersection points, shown in Figure 4. First of all, it calculates two distances between two sets of points. They are calculated from two intersecting points of the small circle with one large circle to the neighboring intersection points of the small circle and another large circle. After calculating two distances, we compare them and select the shorter distance. Finally, this algorithm determines the averaged coordinate of two intersections corresponding to the shorter distance as the MS location.

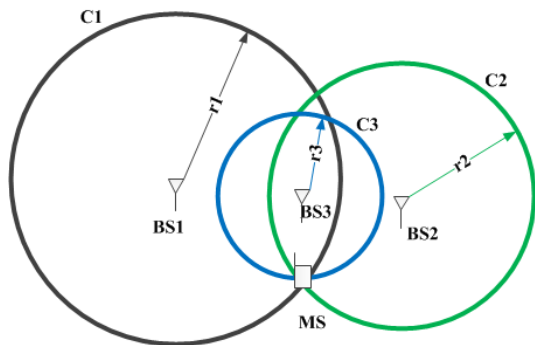


Figure 3. Three intersecting circles to locate MS at a unique point in the specific case

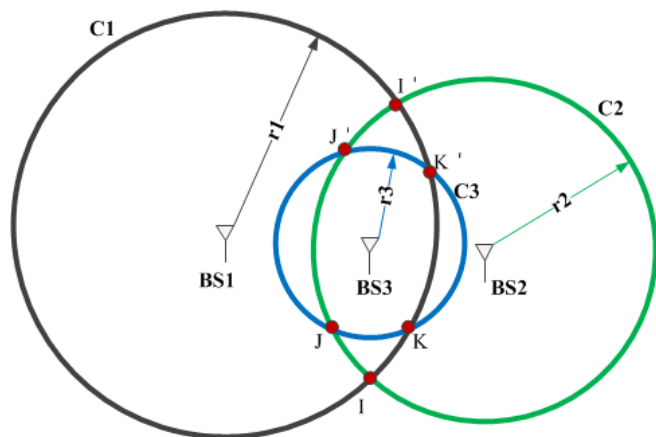


Figure 4. Three intersecting circles with the increased radius in the specific case.

TABLE II. COMPARISON APPROACH OF INTERSECTION DISTANCES

1. Initialize the circles C1, C2, and C3, based on centers of BSs and radiuses equal to estimated distances.
2. Find all six intersections coordinate points I, J, K, I', J', and K'.
3. Take two intersection points of a small circle C3 with two large circles C1 and C2 (J', K') and calculate distance between two points (distance between J' and K').
4. Repeat step 3 for other intersection points (J, K).
5. Compare two distances ((distance J'K') and (distance JK)).
6. Select the shorter distance between two distances.
7. Take two intersection points corresponding to the selected distance.
8. Calculate the averaged coordinate of two intersection points and determine it as the estimated MS location, given by  $\hat{x} = \frac{J'_x + K'_x}{2}$ ,  $\hat{y} = \frac{J'_y + K'_y}{2}$ .

The detail step of comparison approach of intersection distances algorithm has been summarized in Table II.

### III. MODE SELECTION ALGORITHM

The line intersection algorithm and the comparison approach of intersection distances have the good performance for estimating the MS location for the general case and the specific case, respectively. For the optimized location estimation, we should select an algorithm between both according to the case. In this section, we propose the mode selection algorithm for distinguishing the general case and the specific case. If the algorithm selects the general case, it employs the line intersection algorithm for estimating the MS location. However, if it selects the specific case, it employs the comparison approach of intersection distances.

In order to select the case mode, the proposed algorithm calculates four distances related to centers of two large circles and four intersection points of a small circle with two large circles. Each distance is calculated from the center of each large circle to the intersection of the small circle with another large circle, defined as

$$d_{lk} = \sqrt{(x_l - x_k)^2 + (y_l - y_k)^2} \quad l = 1, 2, k = 1, 2, \quad (4)$$

where  $(x_l, y_l)$  is a coordinate of the  $l$ th large circle and  $(x_k, y_k)$  is a coordinate of the intersection point of the small circle with another large circle. Next, the distances related to the  $l$ th large circle are compared to the radius of the  $l$ th large circle;  $d_{11}$  and  $d_{12}$  are compared to  $r_1$  and  $d_{21}$  and  $d_{22}$  are compared to  $r_2$ . If all distances related to the  $l$ th large circle are shorter than the radius of the  $l$ th large circle ( $d_{11}$  and  $d_{12}$  are shorter than  $r_1$  and  $d_{21}$  and  $d_{22}$  are shorter than  $r_2$ ), it selects the specific case mode and employs the comparison approach of intersection distances. Otherwise, it selects the general case mode and employs the line intersection algorithm.

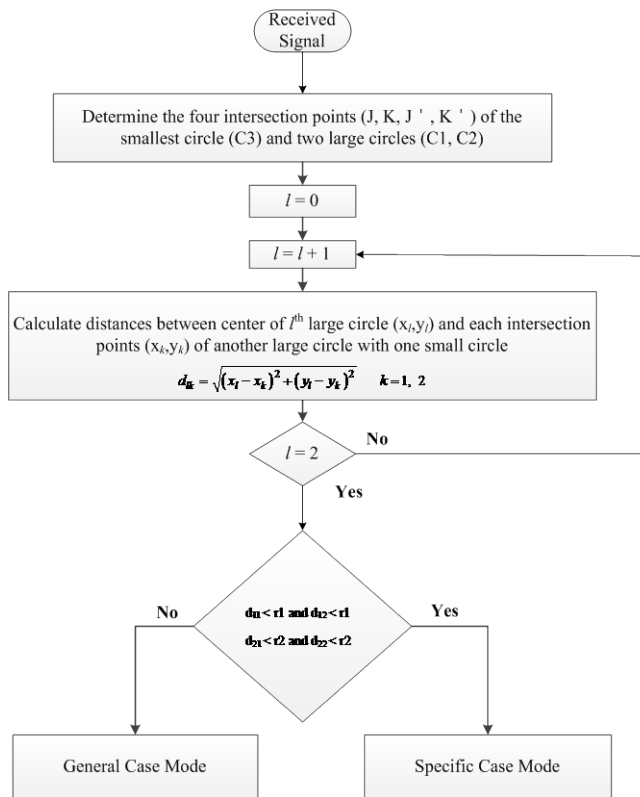


Figure 5. Flow chart for the mode selection algorithm.

TABLE III. MODE SELECTION ALGORITHM

1.	Find the two intersection points (J, J') formed by the small circle C3 intersected with the large circle C2.
2.	Calculate the distances from the center of the large circle C1 ( $x_1, y_1$ ) to the two intersection points (J, J'), given by $d_{1k} = \sqrt{(x_1 - x_k)^2 + (y_1 - y_k)^2}$ $k = 1, 2$ .
3.	Find the two intersection points (K, K') formed by small circle C3 intersected with the large circle C1.
4.	Calculate the distances from the center of the large circle C2 ( $x_2, y_2$ ) to the two intersection points (K, K'), given by $d_{2k} = \sqrt{(x_2 - x_k)^2 + (y_2 - y_k)^2}$ $k = 1, 2$ .
5.	Compare the radius of C1, $r_1$ , to $d_{11}$ and $d_{12}$ .
6.	Compare the radius of C2, $r_2$ , to $d_{21}$ and $d_{22}$ .
7.	a. $d_{11} < r_1$ and $d_{12} < r_1$ , and $d_{21} < r_2$ and $d_{22} < r_2$ ☞ Specific Mode (employing comparison approach of intersection distances)
	b. Otherwise
	☞ General Mode (employing line intersection Algorithm)

Figure 5 shows a flow chart for the proposed mode selection algorithm and Table III summarizes steps of the algorithm. Using this mode selection algorithm, we may get the optimized performance for estimating the location of MS for the general case and the specific case.

#### IV. COMPUTER SIMULATIONS

In this section, we provide computer simulation results to illustrate the location estimation performance for the proposed approach. For the simulation, we consider three fixed BSs and two cases for coordinates of BSs and MS for distinguishing the general case and the specific case:

1. General case: three BSs with coordinates of (-1000, 5000), (6000, -3000), and (-7000, 600).

2. Specific case: three BSs with coordinates of (-3000, 5000), (1500, 3000), and (7000, 600).

The unit of the coordinate is meter (m) and we consider the different carrier frequencies of 50MHz, 100MHz, 500MHz, 1GHz, 5GHz, and 10GHz. Also, we consider two scenarios for the occurrence possibility of the general case and the specific case:

1. First scenario: 90% general case and 10% specific case.

2. Second scenario: 95% general case and 5% specific case.

case.

We assume that the MS location coordinate is randomly chosen with ranges from -100 to 100 and from -500 to 500 for the first case and the second case, respectively.

The performances of the location estimation algorithm are evaluated by the mean square error (MSE). The error between the true MS position and the estimated MS position is defined as

$$Error_{Position} = \sqrt{(x - \hat{x})^2 + (y - \hat{y})^2} \quad (5)$$

and MSE for estimating the MS position is given by

$$MSE_{Position} = E[Error_{Position}^2]. \quad (6)$$

The simulation results of the MSE for the MS location estimation verses frequencies for the first scenario are shown in Figures 6 and 7, for the first and second cases, respectively. In addition, the simulation results of the MSE for the MS location estimation verses frequencies in the second scenario are shown in Figures 8 and 9, for the first and second cases, respectively. From figures, we observe that the MSE of the advanced TOA trilateration based on the mode selection algorithm is lower than the MSE of that based on only the line intersection algorithm. Note that the difference between two curves for the first scenario is larger than the difference for the second scenario, because the occurrence possibility of the specific case in the first scenario is higher than it in the second scenario.

#### V. CONCLUSION

The line intersection algorithm has good location estimation performance for the general case, but it may have the serious location estimation error for the specific case, where a small circle is located in the area of two large circles. Although the comparison approach of intersection distances has worse performance for estimating the MS location compared to the line intersection algorithm in the general case, it has good performance in the specific case. In order to alternately use both algorithms according to the proper case, in this paper, we proposed the mode selection algorithm. The

proposed algorithm compares the distances between the intersection points of the small circle and one large circle to the radius of another large circle. If all distances are shorter than the corresponding radiuses, it selects the specific case mode and employs the comparison algorithm of intersection distances. Otherwise, it selects the general case and employs the line intersection algorithm. The performance for the proposed algorithm was illustrated through computer simulations.

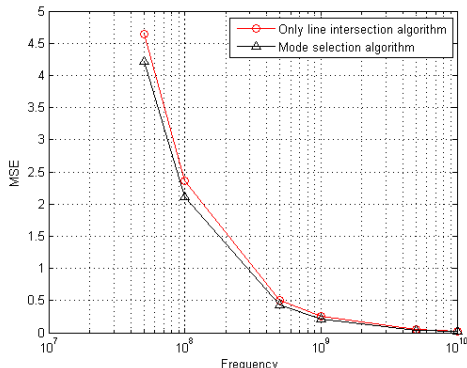


Figure 6. MSE curves for the first scenario for the first case

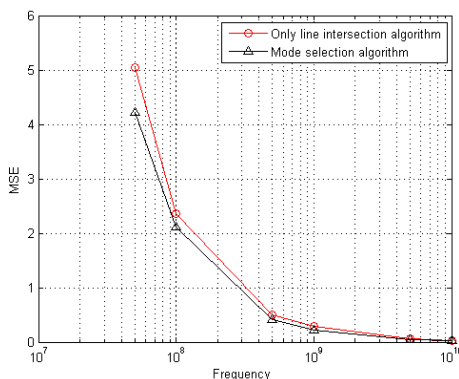


Figure 7. MSE curves for the first scenario for the second case

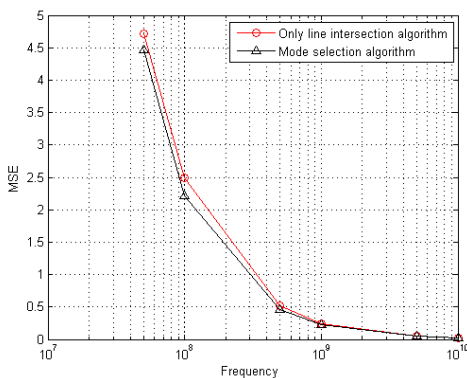


Figure 8. MSE curves for the second scenario for the first case.

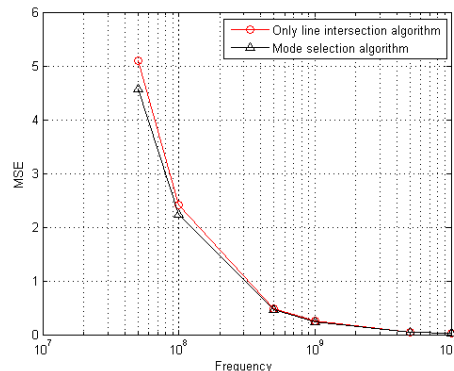


Figure 9. MSE curves for the second scenario for the second case.

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