Receiver Autonomous Integrity Monitoring Performance for Two- Satellites Simultaneous Fault of BeiDou

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***Abstract*— As the probability of two-satellites fault simultaneously increase, the difficulty of fault identification in the integrity monitoring become serious and the traditional Receiver Autonomous Integrity Monitoring (RAIM) may not be able to properly deal with this problem. This paper investigates the impact of two-satellites fault at the same time on integrity parameters. Two factors, two-satellites fault mode and range deviation combination, are introduced to simulate the condition of two satellites fault in BeiDou, and the change of integrity parameters (Horizontal Positioning Error (HPE) and Vertical Positioning Error (VPE)) is discussed. The experiment results show that: under two-satellites fault condition, with range deviation increase, HPE and VPE grow. However, HPE/VPE show little change if the faulty satellite mode contains MEO and the range deviation is less than 100 meters.**

Keywords- Two-Satellites fault; HPE/VPE; Kruskal-Wallis test; Multiple Comparison.

I. INTRODUCTION

RAIM is an important integrity monitoring technique for detecting and identifying the satellite faults based on the redundant observations information received by the user receiver. It has the advantages of no other external equipment, low cost, fast detection speed, and realization convenience and so on. It is one of the integrity monitoring algorithms which is widely used at present [1]-[3].

For the GNSS system, as more and more satellites are lunched for positioning and navigation purpose, the possibility of simultaneous two or more satellites fault increases. As for GPS, in the six years between January 1994 and January 2000, there were 0.9 satellites fall in fault [4], that is, the probability of simultaneous of single satellites, two satellites and triple satellites are 1.0274×10^{-4}/ hours、1.0556×10^{-8}/ hours、1.0845×10^{-12}/ hours, respectively. Therefore, it is of great practical significance to study RAIM aiming at simultaneous multi-satellites fault. However, the traditional RAIM method based on single satellite fault hypothesis may not be suitable for multi-satellites fault condition. This is because the position error caused by simultaneous multi-satellite fault may fail to follow the normal distribution, so that is the traditional RAIM is applied under simultaneous multi-satellites fault condition, it may reduce the performance of fault detection and fault identification [5][10].

In view of the receiver autonomous integrity monitoring under simultaneous multiple-satellites fault situation, scholars have explored many methods [6]-[10].

The above research works of RAIM for multi-satellites fault are based on the improvement of fault detection and fault identification aspect. For fault detection, the works focus on the error model correction, threshold value determination; for fault identification, they mainly include the construction of test statistics and the search for filtering methods. Most of these studies are based on GPS observations.

With the development of BeiDou constellation, its participation in global satellite navigation (GNSS) activities is increasing, and enough attention should be given to the problem of multiple-satellites fault in the constellation. Due to the unique constellation structure, which includes satellites of GEO (Geostationary Earth Orbit), IGSO (Inclined Geo Synchronous Orbit) and MEO (Medium Earth Orbit), fewer studies have been made on the influence of two or more satellites fault on the receiver autonomous integrity monitoring in BeiDou. Therefore, this paper focuses on BeiDou constellation and discusses in depth the impact of BeiDou two-satellites fault on integrity performance.

The content of the article is arranged as follows: Firstly, two-satellites fault positioning model and integrity parameters are introduced; Secondly, the research plan is proposed and the research method based on non-parametric significance test, which includes Kruskal-Wallis test [12] and multiple comparison, is described. Finally, an experiment is designed to simulate the actual two-satellites fault situation. The final section is a summary of the above analysis.

II. POSITIONING MODEL

A. *Positioning model under two-satellites fault*

The linearized positioning model with n observations and m estimated parameters:

$$z = HX + f + \varepsilon \tag{1}$$

where z is $n\times1$ measurement vector;

H is n×m is design metric;

X is $m\times1$ state vector;

$f=(0,\cdots,f_i,\cdots,f_j,\cdots,0)^T$ is fault vector;

$\varepsilon \sim N(0,I_0)$ is $n\times1$ random vector,

I_0 is a diagonal matrix.

If two satellites are faulty at the same time, satellite A

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and satellite B correspond to the ith and jth observation separately, then the equation (1) can be divided into two sub-models, which are the fault measurement sub-model and the fault free measurement sub-model, as shown in equation (2):

$$\begin{bmatrix} \boldsymbol{A}_i^T z \\ \boldsymbol{B}_i^T z \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_i^T H \\ \boldsymbol{B}_i^T H \end{bmatrix} x + \begin{bmatrix} \boldsymbol{A}_i^T f \\ \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{A}_i^T v \\ \boldsymbol{B}_i^T v \end{bmatrix} \qquad (2)$$

where $\boldsymbol{A}_i = \begin{bmatrix} I_2 \\ 0 \end{bmatrix}$， $\boldsymbol{B}_i = \begin{bmatrix} 0 \\ I_{n-2} \end{bmatrix}$，

I_n is a $n \times n$ Diagonal matrix

Estimation of state vector by least square solution is：

$$\hat{\boldsymbol{X}} = \boldsymbol{S}_0 \boldsymbol{z} \qquad (3)$$

where $\boldsymbol{S}_0 = P_0 H^T$ ， $\boldsymbol{P}_0 = (H^T H)^{-1}$ 。

The state estimation error is：

$$\boldsymbol{v}_z = S_0(\varepsilon + f) \qquad (4)$$

$$\boldsymbol{v}_z \sim N(S_0 f, P_0)$$

B. Position Error

Position Error (PE) [11], which is defined in equation (5), is one of the most important parameters in integrity monitoring. It represents the accuracy of positioning solution and directly affects the results of subsequent fault detection and fault identification.

$$\Delta\hat{X} = \hat{X} - X \qquad (5)$$

where $\hat{X}$ is user estimated position, X is user real position. Under user coordination, $\Delta\hat{X} = (\Delta\hat{X}_E, \Delta\hat{X}_N, \Delta\hat{X}_U)$.

Horizontal Position Error（HPE）along east and north direction is defined as in equation (6)：

$$HPE = \sqrt{\Delta\hat{x}_N^2 + \Delta\hat{x}_E^2} \qquad (6)$$

Vertical Position Error（VPE）along up direction is defined as in equation (7):

$$VPE = \sqrt{\Delta\hat{x}_U^2} \qquad (7)$$

III. EXPERIMENTS DESCRIPTION

A. Data Description

The BeiDou II observed data were collected by a Novatel receiver from May 1st, 2017 to May 10th, 2017， Xi'an. BeiDou II system comprises of 14 satellites with GEO satellite labeled from BD 01to BD 05, IGSO satellite labeled from BD 06 to BD 10, MEO satellite labeled from BD 11 to BD 14. The satellite visibility during the observation period is shown in Figure 1. The statistical result of the visible time for each satellite during the observational period shows that BD01- BD 05 was basically visible throughout the observation day, while the visible time of BD06- BD 10 and BD11-14 are 69% and 23% respectively.

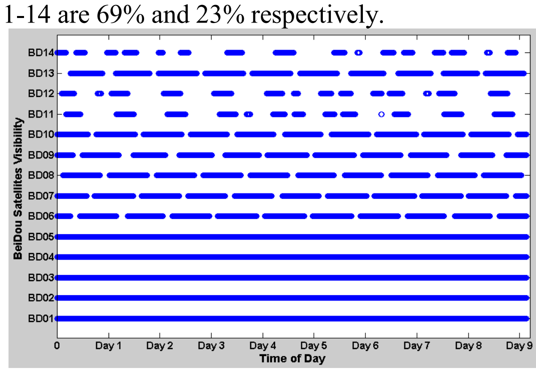

Figure 1. BeiDou Satellite Visibility（1st May, 2017-10th May, 2017，Xi'an）

B. Experiments Scheme

Assuming that the system fault is equivalent to some additional range deviations on one or more receiver observations, therefore, two factors are introduced to simulate the situation of two-satellites fault: (1) two-satellites fault mode [A B]. We selected 6 types of modes, as in Table I, and expressed them by [A B], A is the first fault satellite, and B is the second fault satellite; (2) range deviation combination (a, b). As a satellite fault is considered as the sum of range deviation and its corresponding observation measurement, "a" is described as the range deviation on satellite A and "b" is described as the range deviation on satellite B. The range deviation is set increasing from 50 to 200m with 50m of interval. Table II shows the total of 16 sets of (a, b) combinations.

TABLE I. TWO SATELLITES FAULT MODE

[A B]	GEO	IGSO	MEO
GEO	(01，03)	(01，07)	(O1，12)
IGSO	-	(07，09)	(07，12)
MEO	-	-	(12，12)

TABLE II. RANGE DEVIATION COMBINATIONS UNDER TWO SATELLITES FAULT MODE

（a,b）	FAULT SATELLITE B			
FAULT SATELLITE A	（50,50）	（100,50）	（150,50）	（200,50）
	（50,100）	（100,100）	（150,100）	（200,100）
	（50,150）	（100,150）	（150,150）	（200,150）
	（50,200）	（100,200）	（150,200）	（200,200）

IV. SIGNIFICANCE TEST

In order to investigate data feature of position and integrity under two-satellites fault condition, the non-parametric significance test is introduced and it includes two tests, Kruskal-Wallis (K-W) test and multiple comparisons.

A. K-W test

K-W test is one of the classic methods to infer the significant difference among the test sequences and the overall distribution by using sample data in the case of unknown total variance or little knowledge.

There are two hypotheses for the K-W test. Null hypothesis H_0 is the assumption that all sequences come from the same population and the alternative hypothesis H_1 is the assumption that all sequences come from a different population. For k test sequences A_{N_1}, A_{N_2}, $\cdots$, A_{N_k}, and $N_i = 1,\cdots,k$ is the quantity of ith test sequence. The total quantity of the test sequences is $N = N_1 + N_2 + \cdots + N_{N_k}$. Then, the data of the k test sequences are merged and sorted in ascending order, and the rank of the variable values is obtained. The k sequences of data are combined and sorted, and the serial number values are given, and the serial number values of the k sequences are respectively summed to obtain R_j (j=1, 2, ..., k), and the test statistics H is established on the basis of equation (8):

$$H = \frac{12}{N(N+1)} \sum_{j=1}^{k} \frac{R_j^2}{N_j} - 3(N-1) \tag{8}$$

where $R_{.j} = \sum_{i=1}^{n_j} R_{ij} \, (j = 1,2,\cdots,k)$

After obtaining the test statistics H, the P-value is calculated. The P-value is a probability that the sequences beyond a limitation level under the assumption that the zero hypothesis is correct. It reflects the magnitude of the possibility.

P-value is used to justify whether the hypothesis H_0 is accept or reject by referencing a significant level of 0.05. The significance level refers to the probability that the difference between test sequences caused by sampling error is less than 0.05.

B. Multiple Comparisons

When there is a significant difference among the test sequences based on K-W test, there is reason to consider that all test sequences may not come from the same population. However, this is not to say that there is a difference between any pairs of test sequences. In order to further insight, the difference between the test sequences, multiple comparison is used. For a pair of test sequences A_{N_i} and A_{N_j}, its corresponding confidence interval is constructed [24] in equation (9):

$$\left[\overline{A}_{N_i\cdot} - \overline{A}_{N_j\cdot} - \sqrt{\left(\frac{1}{N_i} + \frac{1}{N_j}\right)}\sigma \times t_{1-\frac{\alpha}{2}}(f_e), \overline{A}_{N_i\cdot} - \overline{A}_{N_j\cdot} + \sqrt{\left(\frac{1}{N_i} + \frac{1}{N_j}\right)}\sigma \times t_{1-\frac{\alpha}{2}}(f_e) \right] \tag{9}$$

where $\overline{A}_{N_i\cdot} = T_h / N_h$, $T_h = \sum_{h=1}^{N_i} A_{N_i,h}$, $A_{N_i,h}$ is the hth element in the ith test sequence, α is significant level, $f_e = N - k$, $\sigma^2 = S_e / f_e$ is Error variance, $S_e = \sum_{i=1}^{k} \sum_{j=1}^{N_j} (A_{N_i,j} - \overline{A}_{N_i\cdot})^2$.

The confidence interval obtained by multiple comparison is used to determine whether any two sets of test sequences come from the same population. When the confidence interval contained zero, it indicated that two test sequences may come from the same population. When the estimated interval did not contain zero, it indicated that two test sequences may not come from the same population.

V. EXPERIMENTAL ANALYSIS

The non-parametric significance test is used to discuss the connection between HPE or VPE and the addition of various range deviation combinations under the same two-satellites fault mode.

Under a certain two-satellites fault mode, the HPE and VPE sequences from the original observations are labeled as $HPE_{(a\ \ b)_0}$ and $VPE_{(a\ \ b)_0}$. HPE and VPE sequences from addition of range deviation on the observations are labeled as $\{HPE_{(a,b)_i}\}$ and $\{VPE_{(a,b)_i}\}$, where $(a\ \ b)_i$, $i = 1,\cdots,16$.

The hypothesis of HPE is:

$$H_0: \quad \mu_{HPE_{(a\ \ b)_0}} = \mu_{\{HPE_{(a,b)_i}\}}$$

$$H_1: \quad \mu_{HPE_{(a\ \ b)_0}} \neq \mu_{\{HPE_{(a,b)_i}\}}$$

The hypothesis of VPE is:

$$H_0: \quad \mu_{VPE_{(a\ \ b)_0}} = \mu_{\{VPE_{(a,b)_i}\}}$$

$$H_1: \quad \mu_{VPE_{(a\ \ b)_0}} \neq \mu_{\{VPE_{(a,b)_i}\}}$$

In the three-dimensional graphics of Figure 2, the X axis represents 6 types of two-satellites fault modes. Y axis represents the 16 sets of range deviation combinations, and the Z axis represents the P-value obtained by the K-W test with a significant level of 0.05. It shows P-values from all tests are significantly less than 0.05. Therefore, there is a significant difference among HPE sequences with introduction of range deviations. That is to say, under two-satellites fault mode, there are one or more sequences which show significant difference with the addition of range deviations, but we fail to know how many HPE sequences are different. The following multiple comparisons are used to identify the HPE sequences with significant difference.

The analysis on VPE is proceeding in the same way.

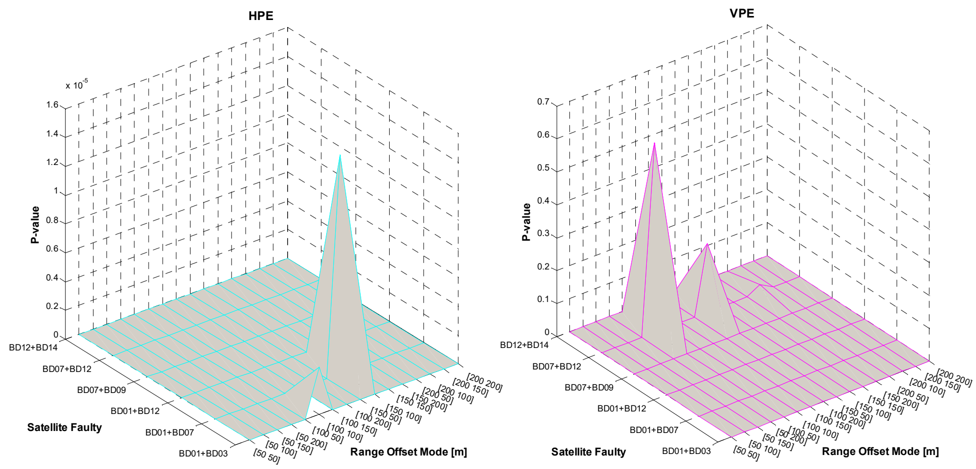

Figure 2. P- values of the K-W test for HPE (left) /VPE (right) under the addition of range deviation combinations and dual-satellite faulty mode (significant level of 0.05)

Table III shows the label scheme in multiple comparisons for HPE. There are 16 comparison groups under each two-satellites fault mode. In each test, $HPE_{(a\ \ b)_0}$ from the original observations is compared with each $\left\{HPE_{(a,b)_i}\right\}$.

TABLE III. K-W TEST SEQUENCE PAIRING STRATEGY FOR THE INFLUENCE OF DIFFERENT RANGE DEVIATION COMBINATIONS ON HPE（LEFT）/VPE(RIGHT)

	[50 50]	[50 100]	[50 150]	……	[200 150]	[200 200]
（a_0 b_0）	GROUP1	GROUP2	GROUP3	……	GROUP15	GROUP16

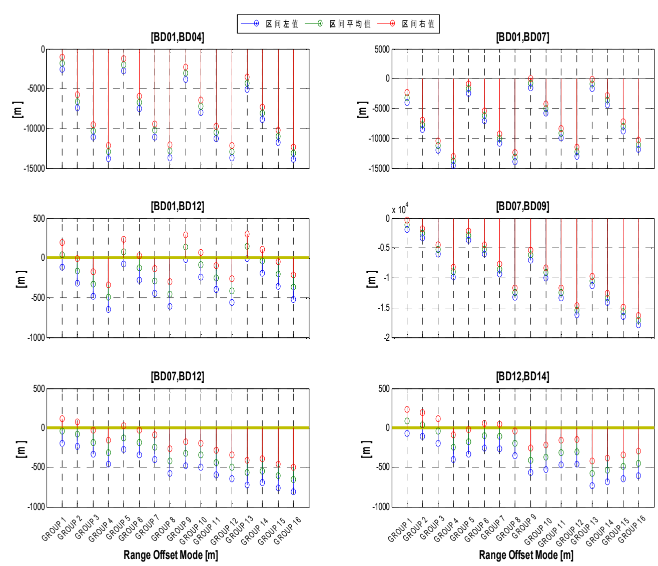

Figure 3. HPE Confidence interval of multiple comparisons before and after introducing range deviation combination.

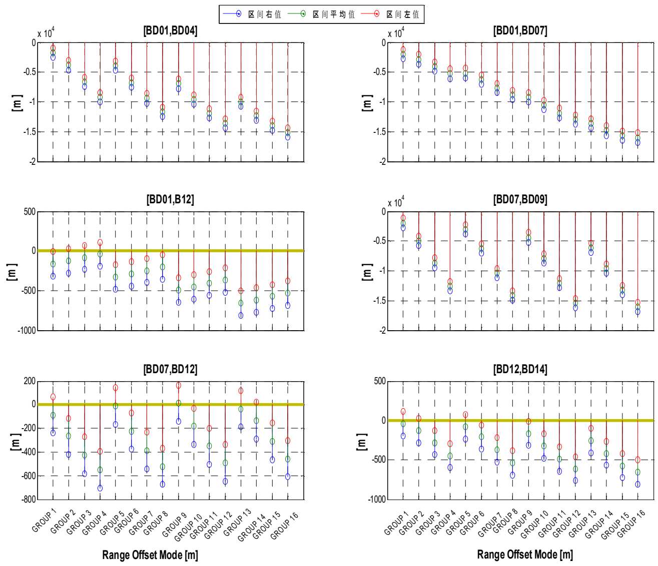

Figure 4. VPE Confidence interval of multiple comparisons before and after introducing range deviation combination.

As shown in Figure 3 and Figure 4：

1) Under three types of two-satellites fault mode, [BD01, BD04], [BD01 BD07] and [BD07 BD09] HPEs show significant difference with addition of range deviation.
2) There is little difference of HPE among [BD01 BD12], [BD07 BD12] and [BD12 BD14]. It is worthy to know that these three types of modes include MEO satellite, such as BD12 OR BD 14.

TABLE IV. RANGE DEVIATION COMBINATIONS FOR TWO-SATELLITES FAULTY MODE WITH NO SIGNIFICANT DIFFERENCE IN MULTIPLE COMPARISONS FOR HPE (LEFT) /VPE (RIGHT)

[BD01 BD12]	[BD07 BD12]	[BD12 BD14]
(50 50)	(50 50)	(50 50)
(100 50)	(50 100)	(50 100)
(100 100)	(100 50)	(50 150)
(150 50)		(100 100)
(150 100)		(100 0)
(200 50)		
(200 0)		

Furthermore, Table IV shows that when the range deviation combination is (50, 50) m, there is hardly any difference among the HPE comparison groups. Specifically, in [BD01 BD12], after the addition of 200m of range deviation on BD01, there is little difference of HPEs against the condition that no additional range deviation. The difference becomes significant when 100m addition of range deviation for BD12.

As for [BD07 BD12], as long as the range deviation is less than 100 meters, there is hardly any difference between the test sequences.

In [BD12 BD14], although both BD12 and BD14 are MEO satellites, it shows that the range deviation combination does not affect them in the same way. That is, when the range deviation combination is (50 100)m, HPE sequences shows a significant difference while HPE sequences have little difference when the range deviation (100 50)m is introduced.

VI. CONCLUSION AND FUTURE WORK

With the development of the Beidou system, the probability of simultaneous failures of two satellites or multiple satellites increases. Based on two-satellites fault condition, this paper discusses the impact of two satellites faults on the integrity parameters HPE/VPE. The result shows that: under two-satellites fault, the range deviation growth leads to the increase of HPE or VPE. However, if the satellite fault mode contains MEO and the range deviation is less than 100m, the effect on HPE/VPE is not significant.

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