Heterogeneous Migration Paths to High Bandwidth Home Connections - a Computational Approach

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***Abstract*—Most telecom operators have to plan the migration of their existing copper networks to full or hybrid fibre networks, to offer their clients the bandwidth they require. This paper proposes methods to optimise this migration path, heterogeneously per central office area, using geometric models as input. The methods result in a detailed migration path that meets a required bandwidth coverage, installation capacity and/or budget constraint. To solve the optimisation problem in an efficient way, both a problem-based solution method and a simulated annealing approach are tested for scalability of the problem solving. As the data used for the migration path optimisation is in practice hard to gather, the use of geometric modelling is proposed. This modelling approach leads to the optimal migrating path, estimating the total initial investment of a migration step using only two simple parameters per Central Office area.**

Index Terms*—*Access networks; Migration Optimisation; Geometric Models.

I. Introduction

Broadband internet is becoming a common utility service. Using connected electronic devices in and outside our homes, we use more and more data and demand connectivity 24/7. The used services are asking more bandwidth due to the integration of video into numerous services. Most of the home connections, access networks and systems offered by telecom operators are not prepared for this, because incumbent operators mostly use copper telecommunication networks offering ADSL (Asymmetric Digital Subscriber Line) or VDSL (Very High Bitrate Digital Subscriber Line) techniques as service . Digital subscriber line (DSL) is a family of technologies used to transmit digital data over copper lines. The operators have to make the costly step to Fibre to the Cabinet (FttCab), Fibre to the Curb (FttCurb) or, even more costly, the full step to Fibre to the Home (FttH), Fibre near the home (FntH) or Fibre to the Air (FttA). Bringing the network to the next step we call a migration step, as introduced in [1]. An example of FntH and FttA is a wireless home connection or a Hybrid fibre-wireless (FiWi) access network, where fibre is brought to a location near the homes, e.g., street lights [2], and the remaining distance is covered by WiFi or WiMax [3] [4]. However, in many countries, the roll out of all these fibre connections will take too long to compete with the cable TV operators active in those countries, who can offer the required bandwidth using DOCSIS, Data Over Cable Service Interface Specification, on their Hybrid Fibre Coaxial (HFC) networks at this moment. This urges the operators to take intermediate steps, such as FttCurb, and to think about the optimal migration strategy.

The incumbent telecom operators can choose between various topology types to offer. In this paper, the term ‘topology’ is used for the way the physical fibres and equipment are designed. It comprises the question where to deploy fibres, where to deploy copper and where the active or passive equipment should be placed. Each topology can run multiple technologies. For example, in the ‘Full Copper’ topology, the operator offers the services from the Central Office. The operator still can choose to offer ADSL or VDSL (containing here all VDSL based technologies such as VDSL, VDSL2, Vectored VDSL2, Vplus etcetera) technology for this service. In this paper, four topology types are distinguished (see Fig. 1):

1) Full Copper: services are offered from the Central Office (CO) over a copper (twisted pair) cable, using DSL techniques.
2) Fibre to the Cabinet (FttCab): the fibre connection is extended to the cabinet. From the cabinet, the services are offered over the copper cable, using DSL or G.Fast techniques.
3) Hybrid Fibre to the Home (Hybrid FttH): services are offered from a Hybrid FttH Node, which is connected by fibre, close to the customer premises, in the street, or in the building. Here again, VDSL and G.Fast techniques can be offered.
4) Full Fibre to the Home (Full FttH): the fibre connection is brought up to the customer premises.

If the operator starts with a Full Copper topology in a certain area, he has to decide on the next step: bringing the fibre connection all the way to the customers or use an intermediate step, where he brings the fibre closer to the customer, e.g., FttCab. Note that the operator can have a heterogeneous network, where in different areas a different topology is deployed and a different starting position for

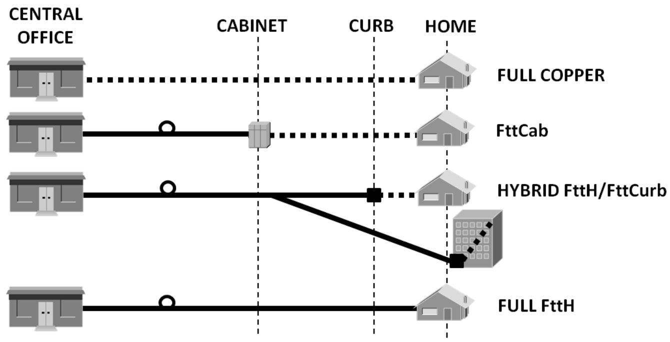

Figure 1. Four topologies.

migration is found. To make in a certain area the decision mentioned before, the operator has to look at the pros and cons of all the options. For example, the deployment of FttCab can be much faster than Full FttH, as it requires less digging, the last part of the connection from the street to the access node in the house does not have to be installed, and it meets the growing bandwidth demand for now and the near future. If, in future, this demand exceeds the supplied bandwidth, the remaining part to the residence can be connected with Full Fibre or using Hybrid Fibre as extra intermediate step. If the demand does not exceed the supplied bandwidth, for example, it reaches some level of saturation, no further migration is needed, saving a lot of investments. However, when Full FttH is the expected final solution, using intermediate steps would incur investment and installation costs that might be lost and not reused.

This decision can be made on strategic level, for a bigger region or a whole country, or more tactical/operational within a region. In this paper, the option that the operator can decide per Central Office area which topology or technology to offer is considered. This means that the operator is offering broadband as a service, instead of offering for example FttH as a service. If an operator decides on the topology or technique per Central Office area and per period (e.g., year), he can develop a detailed migration path that meets, for example, a bandwidth coverage in a larger area. This option is called a heterogeneous optimal migration path in contrast to a homogeneous optimal migration path, where one migration path is used for all Central Office areas within the bigger region. This is the first part where the novelty of this paper is in. Up to now, other papers only considered uniform migration paths or single migration paths.

The second part where this paper is novel, is the data used for the migration path optimisation. To estimate the costs of a topology and the migration from one topology to another topology, for each migration an optimal planning should be made. We propose to solve this by using the geometric modelling, as presented in [5] [6].

Concluding, in this paper, we present a methodology that can be used by operators to design their heterogeneous topology migration path from Full Copper to Full FttH, meeting their business requirements. First, we start with a literature survey on related models in Section II. In Section III, a model is presented to optimise the heterogeneous migration paths, where the complexity of the model is discussed in Section IV. In Section V, a method is presented to gather the input for the migration path optimisation using Geometric models. In Section VI, solution methods are presented in order to get a solution to the problem in reasonable time. Next, in Section VII, the optimisation method is demonstrated by a case study and the scalability of finding a solution is shown by computational results. Finally, in Section VIII, some conclusions are presented.

II. Literature

Migration within telecommunication networks is a topic in many Techno-Economical studies. In these studies, the economic sanity of some choices are investigated. The European projects IST-TONIC [7] and CELTIC-ECOSYS [8] resulted in various upgrade or deployment scenarios for both fixed and wireless telecommunication networks, which was published in [9] and [10]. A major question in these studies is when to make the decision to roll out a FttC/VDSL network or a Full FttH network. Based on demand forecasts, it was shown that it is profitable to start in dense urban areas, wait for five years and then decide to expand it to the other urban areas. With the use of real option valuation, the effect of waiting is rewarded to identify the optimal decision over time. In [11] and [12], the OASE approaches are presented for more in depth analysis of the FttH total cost of ownership (TCO) and for comparing different possible business models both qualitatively and quantitatively.

The work of Casier [13] presents the techno-economic aspects of a fibre to the home network deployment. First, he considers all aspects of a semi-urban roll-out in terms of dimensioning and cost estimation models. Next, the effects of competition are introduced into the analysis.

The work in [14] presents a multi-criteria model aimed at studying the evolution scenarios to deploy new supporting technologies in the access network to deliver broadband services to individuals and small enterprises. This model is based on a state transition diagram, whose nodes characterise a subscriber line in terms of service offerings and supporting technologies. This model was extended for studying the evolution towards broadband services and create the optimal path for broadband network migration. A similar kind of model is presented in [15] and [16], where also an optimal strategy is proposed using a dynamic migration model. They study the best migration path including investments (capital expenditures, CapEx) and operational expenditures (OpEx) and revenues. Several fixed access technologies are considered as intermediate steps. A more recent study [17], proposes several migration strategies for active optical networking from data plane, topology, and control plane perspectives, and investigates their impact on the total cost of ownership. However, these migration strategies are not optimised.

Finally, our own previous work was about the benefits of a migration path as alternative for the direct step from Full Copper to FttH [18], and a Techno-Economic model [19] that can calculate the effect on market share, revenues, costs and earnings of offering different topologies and technologies in access networks (in migration).

As said earlier, all these approaches only consider uniform or single migration paths and do not include the possibility of using geometric models as input.

III. Migration model

A migration path is here defined as a path from one topology/technique combination to a destination topology/technique, possibly using other topology/technique combinations as intermediate steps. Analogue to [15], we use a figure to clarify the idea, see Fig. 2.

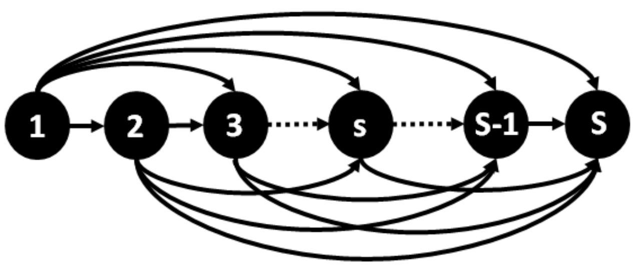

Figure 2. Migration paths.

Each node here is a topology/technique combination. One can choose a path from node 1, typically Full Copper/ADSL, to node S, typically FttH. So examples for the paths are: Full Copper/ADSL to FttH, Full Copper/ADSL to FttCab/VDSL to FttH, Full Copper/ADSL to FttCab/VDSL to FttCurb/G.Fast, etcetera. The focus in this paper is on an area that consists of multiple Central Offices, which is the location of the switching equipment, to which subscriber home and business lines are connected on a local loop, for example, a city or a district. The goal of the operator is to offer in this district a certain bandwidth coverage (per year), given a budget (per year) and possibly other constraints. A bandwidth coverage can be a single value, e.g., 'I want to offer 100 Mb/s in 2017', or a distribution over various bandwidth values in a number of years. An example of this distribution over years is presented in Table I. In the table is stated that in 2018 (at least) 60% of the houses need to have (at least) 100 Mb/s, at least 40% of the houses need to have (at least) 200 Mb/s and (at least) 10% need (at least) 300Mb. The percentages do not add up to 100% as they are exceedance probabilities. If all houses have a connection that offers 500 Mb/s the bandwidth coverage demand is met, obviously.

Now, the problem can be defined as an Integer Programming Problem. The notation that is used is presented in Table II. First, the objective function is defined as:

$$\min \sum_{i \in I} \sum_{j \in I} \sum_{l \in L} \sum_{t \in T} C_{ijl} x_{ijlt}. \quad (1)$$

TABLE I. COVERAGE GOAL.

Year	100 Mb/s	200 Mb/s	300 Mb/s
2018	60%	40%	10%
2021	80%	60%	20%
2024	90%	80%	40%

TABLE II. DEFINITIONS

Notation		Description
I	=	set of topologies/technologies;
L	=	set of locations, here CO areas;
T	=	set of time periods;
D	=	set of distances, e.g.,{200m, 400m, 600m} ;
x_{ijlt}	=	$\begin{cases} 1 & \text{if migration takes place from technology i} \in I \text{ to} \\ & \text{j} \in I \text{ in year } t \in T \text{ for location } l \in L \\ 0 & \text{otherwise} \end{cases}$
y_{ilt}	=	$\begin{cases} 1 & \text{if technology i} \in I \text{ is active on time } t \in T \\ & \text{at location l} \in L \\ 0 & \text{otherwise} \end{cases}$
C_{ijl}	=	migration costs for going from technology $i \in I$ to $j \in I$ at location $l \in L$
H_{ijl}	=	required installation capacity for migrating from technology $i \in I$ to $j \in I$ at location $l \in L$
O_{ilt}	=	operation costs when technology $i \in I$ is active at time $t \in T$ at location $l \in L$
R_{ild}	=	number of houses reached by technology $i \in I$ within distance $d \in D$ for location $l \in L$;
RT_l	=	total number of premises at location $l \in L$;
G_{td}	=	requested percentage of premises to be reached within distance $d \in D$ T time $t \in T$;
B_t	=	maximum budget available for time $t \in T$;
IC_t	=	installation capacity available for time $t \in T$;

This objective function minimises the total costs (CapEx) for the migration under the following constraints:

$$\sum_{i \in I} \sum_{j \in I} x_{ijlt} \leq 1, \ \forall\, t \in T, l \in L \quad (2)$$

$$\sum_{i \in I} y_{ilt} = 1, \ \forall\, l \in L, t \in T \quad (3)$$

$$x_{ijlt} \geq \frac{1}{2}(y_{jlt} - y_{jlt-1}) - \frac{1}{2}(y_{ilt} - y_{ilt-1}) - \frac{1}{2} \quad \forall\, i, j \in I, l \in L, t \in T \quad (4)$$

$$\frac{\sum_{i \in I} \sum_{l \in L} R_{ild} \cdot y_{ilt}}{\sum_{l \in L} RT_l} \geq G_{td}, \ \forall\, t \in T, d \in D \quad (5)$$

$$x_{ijlt}, y_{ilt} \in \{0, 1\}, \ \forall\, i, j \in I, l \in L, t \in T \quad (6)$$

This model will be called the base model. Constraint (2) makes sure that there is at most 1 migration step per year per location. Constraint (3) makes sure that each location has exactly 1 topology each year. Constraint (4) creates the migration steps. The right term can only be greater than zero if (and only if) $(y_{jlt} - y_{jlt-1}) = 1$ and $(y_{ilt} - y_{ilt-1}) = -1$, which indicates that there is a transition from technology i to technology j. Constraint (5) makes sure that the required bandwidth is delivered.

An alternative objective function is realised when adding the operational cost, or OpEx. This alters the objective function in:

$$\min \sum_{i\in I}\sum_{j\in I}\sum_{l\in L}\sum_{t\in T} C_{ijl}x_{ijlt} + \sum_{i\in I}\sum_{l\in L}\sum_{t\in T} O_{ilt}y_{ilt} \quad (7)$$

An other alternative model, called the extended model, is the model in which there exists a budget constraint per time period and a constraint for the installation capacity. In this formulation, the budget constraints are hard and the gap between the realised and demanded bandwidth per year is minimised.

$$\min \sum_{t\in T}\sum_{d\in D} \max\left(0, G_{td} - \frac{\sum_{i\in I}\sum_{l\in L} R_{ild}\cdot y_{ilt}}{\sum_{l\in L} RT_l}\right), \quad (8)$$

For the extended model, the following constraints should hold:

$$\begin{aligned}
\sum_{i\in I}\sum_{j\in I} x_{ijlt} &\leq 1,\ \forall\, l \in I, t\in T && (9)\\
\sum_{i\in I} y_{ilt} &= 1,\ \forall\, l \in L, t\in T && (10)\\
\frac{1}{2}(y_{jlt} - y_{jlt-1}) - \frac{1}{2}(y_{ilt} - y_{ilt-1}) - \frac{1}{2} &\leq x_{ijlt}, \\
&\quad \forall\, i,j \in I, l\in L, t\in T && (11)\\
\sum_{i\in I}\sum_{j\in I}\sum_{l\in L} c_{ijl}x_{ijlt} &\leq B_t,\ \forall\, t\in T && (12)\\
\sum_{i\in I}\sum_{j\in I}\sum_{l\in L} h_{ijl}x_{ijlt} &\leq IC_t,\ \forall\, t\in T && (13)\\
x_{ijlt}, y_{ilt} &\in \{0,1\}, \\
&\quad \forall\, i,j \in I, l\in L, t\in T && (14)
\end{aligned}$$

where (12) and (13) are added as budget and installation capacity constraints. This problem is no longer an ILP, as the objective is not linear. However, it can be linearised by introducing the variable z_{td} with $t \in T, d \in D$. Furthermore, the following constraints for z_{td} are added to the model:

$$\begin{aligned}
z_{td} &\geq 0 && \forall\, d\in D, t\in T \quad (15)\\
z_{td} &\geq G_{td} - \frac{\sum_{i\in I}\sum_{l\in L} R_{ild}\cdot y_{ilt}}{\sum_{l\in L} RT_l} && \forall\, d\in D, t\in T \quad (16)
\end{aligned}$$

As a result, the objective function changes to:

$$\min \sum_{t\in t}\sum_{d\in D} z_{td}. \quad (17)$$

Moreover, z_{td} does not have to be integer. Summarising, the extended model used for creating an optimal solution is:

$$\begin{aligned}
\min \sum_{t\in T}\sum_{d\in D} z_{td} & && (18)\\
\text{s.t.} \sum_{i\in I}\sum_{j\in J}\sum_{l\in L} C_{ijl}x_{ijlt} &\leq B_t,\ \forall\, t\in T && (19)\\
\sum_{i\in I}\sum_{j\in J}\sum_{l\in L} H_{ijl}x_{ijlt} &\leq IC_t,\ \forall\, t\in T && (20)\\
\sum_{i\in I}\sum_{j\in J} x_{ijlt} &\leq 1,\ \forall\, l\in L, t\in T && (21)\\
\sum_{i\in I} y_{ilt} &= 1,\ \forall\, l\in L, t\in T && (22)\\
\frac{1}{2}(y_{jlt} - y_{jlt-1}) - \frac{1}{2}(y_{ilt} - y_{ilt-1}) - \frac{1}{2} &\leq x_{ijlt}, \\
&\quad \forall\, i,j\in I, l\in L, t\in T && (23)\\
z_{td} &\geq G_{td} - \frac{\sum_{i\in I}\sum_{l\in L} R_{ild}\cdot y_{ilt}}{\sum_{l\in L} RT_l}, \\
&\quad \forall\, d\in D, t\in T && (24)\\
z_{td} &\geq 0,\ \forall\, d\in D, t\in T && (25)\\
x_{ijlt}, y_{ilt} &\in \{0,1\}, \\
&\quad \forall\, i,j\in I, l\in L, t\in T. && (26)
\end{aligned}$$

IV. Complexity

In this section, the complexity of the base model and the extended model are discussed and it is shown that both models are NP-hard.

A. Base model

The Single Source Capacitated Facility Location Problem (SSCFLP) is NP-hard and can be reduced to the base model of the Migration of Fibre problem. In this problem, a number of facilities should be located, whereby each customer is fully assigned to a facility at minimum cost, such that the demand of each customer is served and a facility does not supply more than his capacity. This can be described as follows [20]:

$$\begin{aligned}
\min \sum_{i\in Q}\sum_{j\in J} V_{ij}x_{ij} + \sum_{j\in J} F_j y_j & && (27)\\
\text{s.t.} \sum_{j\in J} x_{ij} &= 1 && \forall i \in Q \quad (28)\\
x_{ij} &\leq y_j \\
& && \forall i\in Q, j\in J \quad (29)\\
\sum_{i\in Q} K_i x_{ij} &\leq S_j y_j && \forall j\in J \quad (30)\\
x_{ij}, y_i &\in \{0,1\} \\
& && \forall i\in Q, j\in J, \quad (31)
\end{aligned}$$

where I is the set of customers, J is the set of facilities, V_{ij} are the costs for assigning customer $i \in I$ to facility $j \in J$ and F_j are the costs for opening facility $j \in J$. Furthermore,

K_i is the demand of customer $i \in I$ and S_j is the capacity of location $j \in J$. It holds that variable x_{ij} is equal to 1 if customer $i \in I$ is assigned to facility $j \in J$. Otherwise, this variable is equal to 0. The variable y_j is equal to 1 if facility $j \in J$ is opened. Otherwise, this variable is equal to 0.

For the reduction of the SSCFLP to the base problem, firstly, one instance of the base problem is given. Assume there is one location l, one time period $t \in T$ and one distance $d \in D$. By this, the base problem can be reduced to:

$$\min \sum_{i=i_0} \sum_{j \in I} C_{ij} x_{ij} \tag{32}$$

$$\text{s.t.} \sum_{j \in I} x_{ij} \leq 1 \quad \forall\, i = i_0 \tag{33}$$

$$\sum_{j \in I} y_j = 1 \tag{34}$$

$$\frac{1}{2}(y_j - Y_j) - \frac{1}{2}(y_i - Y_i) - \frac{1}{2} \leq x_{ij} \quad \forall i = i_0, j \in I \tag{35}$$

$$\frac{\sum_{j \in I} R_j \cdot y_j}{RT} \geq G \tag{36}$$

$$x_{ij}, y_j \in \{0,1\} \quad \forall\, i = i_0, j \in I. \tag{37}$$

Here, i_0 denotes the start state, namely the combination i of a technology and a topology at time $t = 0$. So, Y_i is equal to 1 for i equal to the technology and topology combination at time period $t = 0$ and zero otherwise. Note that Constraint (5) does not force that $x_{ij} = 1$ for $i = j$ when no migration takes place. However, having $x_{ij} = 1$ for $i = j$ does not affect the objective function, because the migration costs for migrating to the same technology and topology combination i are zero, as there is actually no migration happening. As a result, we can change Constraint (5) to:

$$x_{ij} \geq Y_i + y_j - 1, \qquad \forall\, i = i_0, j \in I. \tag{38}$$

In this equation, it is forced that $x_{ij} = 1$ for $i = j$ when no migration takes place. We know that $Y_{i_0} = 1$, because i_0 is the start state, thus Constraint (38) can be changed to:

$$x_{ij} \geq y_j, \qquad \forall\, i = i_0, j \in I. \tag{39}$$

In the base model, Constraint (33) has got an inequality sign and not an equality sign due to the fact that there is no time period $t = -1$ before the start state, so there is no migration possible from $t = -1$ to $t = 0$, and thus, for the start state $t = 0$ it holds that $x_{ijl0} = 0$ for all $i, j \in I$ and $l \in L$. In the ILP described above, we only have one time period $t \in T$, so we can change the inequality sign in Constraint (33) to an equality sign:

$$\sum_{j \in I} x_{ij} = 1, \qquad \forall\, i = i_0. \tag{40}$$

As a result of Constraint (34) and (40), we can flip the inequality sign in Constraint (38), because this does not affect the relation between the y_j and $x_{i_0 j}$, which should be equal to each other:

$$x_{ij} \leq y_j, \qquad \forall i = i_0, j \in I. \tag{41}$$

From Constraint (34), we know that exactly one technology and topology combination $j \in I$ should be active in the considered time period. We can replace this constraint by adding the following part to the objective function:

$$\sum_{j \in I} U_j y_j, \tag{42}$$

where it holds that $U_j = U$ for all $j \in I$ and $U > \max_{\forall i, j \in I} C_{ij}$. As a result, the ILP becomes:

$$\min \sum_{i=i_0} \sum_{j \in I} C_{ij} x_{ij} + \sum_{j \in I} U_j y_j \tag{43}$$

$$\text{s.t.} \sum_{j \in I} x_{ij} = 1 \quad \forall\, i = i_0 \tag{44}$$

$$x_{ij} \leq y_j \quad \forall\, i = i_0, j \in I \tag{45}$$

$$\frac{\sum_{j \in I} R_j \cdot y_j}{RT} \geq G \tag{46}$$

$$x_{ij}, y_i \in \{0,1\} \quad \forall\, i, j \in I. \tag{47}$$

From Constraint (44), (45) and the fact that only one technology and topology combination $j \in I$ could be active, it follows that we can remove the sum in Constraint (46), by adding $x_{i_0 j}$ at the right side of the inequality sign. This is because it must hold that the active technology and topology combination j after migrating fulfils the bandwidth demand G. This results in the following ILP:

$$\min \sum_{i=i_0} \sum_{j \in I} C_{ij} x_{ij} + \sum_{j \in I} U_j y_j \tag{48}$$

$$\text{s.t.} \sum_{j \in I} x_{ij} = 1 \quad \forall\, i = i_0 \tag{49}$$

$$x_{ij} \leq y_j \quad \forall\, i = i_0, j \in I \tag{50}$$

$$\frac{R_j}{RT} \cdot y_j \geq \sum_{i=i_0} G_i x_{i_0 j} \quad \forall\, j \in I \tag{51}$$

$$x_{ij}, y_i \in \{0,1\} \quad \forall\, i, j \in I, \tag{52}$$

where $G_i = G$ for all $i \in I$.

The values C_{ij}, U_j, $\frac{R_j}{RT}$ and G_i for all $i, j \in I$ correspond to the SSCFLP values V_{ij}, F_j, S_j and K_i for all $i \in Q$ and $j \in J$, respectively. Moreover, i_0 is the set of customers Q and the set of facilities J is equal to the set I of technology and topology combinations. This shows that the SSCFLP is a special case of the base problem and leads to the conclusion that the base problem is at least as hard as the SSCFLP. The SSCFLP is NP-hard [20], and thus, the base problem is also NP-hard.

B. Extended model

The Multiple Constraint Knapsack Problem is NP-hard and can be reduced to the extended model of the Migration of Fibre problem. In this problem, a set of items, each with a weight and value, could be packed once into a knapsack. The objective is to determine which item to include in the knapsack, to maximise the total profit and without exceeding the knapsack constraints. This can be described as follows [21]:

$$\max \sum_{i \in I} P_i y_i \tag{53}$$

$$\text{s.t.} \sum_{i \in I} A_{ji} y_i \leq W_j \qquad \forall j \in M \tag{54}$$

$$y_i \in \{0,1\} \qquad \forall i \in I, \tag{55}$$

where the sets of items is given by set I and the set of knapsack constraints is given by set M with corresponding capacities W_j with $j \in M$. The required capacity of item i for knapsack constraint j is A_{ji} with $j \in M, i \in I$. The value of item i is denoted by P_i and y_i is equal to 1 if item i is in the knapsack and otherwise this variable is equal to 0.

Similarly, for the reduction of the Multiple Constraint Knapsack problem to the extended problem, firstly, one instance of the extended problem is given. Assume there is one location $l \in L$, one time period $t \in T$ and one distance $d \in D$. By this, the extended model is reduced to:

$$\min \max \left(0, G - \sum_{i \in I} \frac{R_i}{RT} \cdot y_i \right) \tag{56}$$

$$\text{s.t.} \sum_{i \in I} \sum_{j \in I} C_{ij} x_{ij} \leq B \tag{57}$$

$$\sum_{i \in I} \sum_{j \in I} H_{ij} x_{ij} \leq IC \tag{58}$$

$$\sum_{i \in I} \sum_{j \in I} x_{ij} \leq 1 \tag{59}$$

$$\sum_{i \in J} y_i = 1 \tag{60}$$

$$\frac{1}{2}(y_j - Y_{j_0}) - \frac{1}{2}(y_i - Y_{i_0}) - \frac{1}{2} \leq x_{ij} \qquad \forall i, j \in I \tag{61}$$

$$x_{ij}, y_i \in \{0,1\} \qquad \forall i, j \in I. \tag{62}$$

Again, i_0 denotes again the start state, namely the combination i of a technology and a topology at time $t = 0$. The objective function is a max-min function. However, it is possible to modify the objective function to a maximisation function. Since there is only one location, the objective function can be changed to maximising the bandwidth for this location. The new objective function is defined as:

$$\max \sum_{i \in I} \frac{R_i}{RT} \cdot y_i. \tag{63}$$

Furthermore, the amount of variables can be reduced. This is possible, because there is only one location, one time period and the start state is known. Therefore, x_{ij} can be replaced by y_i. As a result, C_{ij} and H_{ij} are respectively changed to C_i and H_i, and Constraint (59) and (61) become superfluous. Without loss of generality, the equality sign in Constraint (60) can be changed to a "less than or equal to" sign, because the optimal solution will never be $y_i = 0$, for all $i \in I$, due to the used objective function and positive values of $\frac{R_i}{RT}$. Furthermore, it holds that $C_i = 0$ and $H_i = 0$ for $i \in I$ equal to the start state. Summarising, the described instance of the Migration of Fibre problem becomes:

$$\max \sum_{i \in I} \frac{R_i}{RT} \cdot y_i \tag{64}$$

$$\text{s.t.} \sum_{i \in I} C_i y_i \leq B \tag{65}$$

$$\sum_{i \in I} H_i y_i \leq IC \tag{66}$$

$$\sum_{i \in I} y_i \leq 1 \tag{67}$$

$$y_i \in \{0,1\} \qquad \forall i \in I. \tag{68}$$

The budget B, installation capacity IC and 1 correspond to the knapsack capacities W_1, W_2 and W_3, respectively. Furthermore, C_i corresponds to A_{1i} for all $i \in I$, H_i corresponds to A_{2i} for all $i \in I$, and A_{3i} is equal to 1 for all $i \in I$. Lastly, $\frac{R_i}{RT}$ is equal to P_i for all i, thus the Multiple Constraint Knapsack problem is a specific case of the extended problem. This leads to the conclusion that the extended problem is at least as hard as the Multiple Constraint Knapsack problem. The Multiple Constraint Knapsack problem is NP-hard [21], thus, the extended problem is also NP-hard.

V. Input from Geometric model

In the previous section, two parameters are used that are not that easy to obtain, namely c_{ijl}, the cost for migrating from technology i to j at location l, and R_{ild}, the number of premises reached by technology i within d meter at location l. To get the value of these parameters, for each migration an optimal planning should be made. We introduce an alternative for this problem by using the outcomes of geometric modelling, as presented in [5] and [6]. This means that we start by a simple set of parameters per (currently) active node: the total cable length (D) and the capacity of this node (n), which equals the number of premises connected. Note that in this section d and D mean something different, using the notation of [6], than in the model of the previous sections. As is shown in [6], from these parameters the geometric density of the premises can be derived. With this geometric density, we can estimate the number of new active locations that a next technology needs in this area to achieve a certain distance coverage, and consequently, the bandwidth coverage. From this number of active elements, the costs of the migration can be estimated. Next, using the same density, also the cable and digging distances to connect those new active elements can be estimated.

To illustrate this approach, think of an area, currently equipped with VDSL2, that contains $n_1 = 1,000$ houses. The given total cable length equals $D_1 = 875,000$ meter. Now, the parameter d, which indicates the house density of the area, expressed in the (average) width of the premises, can be derived by solving (using $s_1 = \sqrt{n_1}$):

$$d = \frac{D_1}{2 \cdot s_1 \cdot \lceil \frac{1}{2} s_1 \rceil \cdot \lfloor \frac{1}{2} s_1 \rfloor}, \tag{69}$$

resulting in $d = 57.7$ for the given example. Let us assume that in the next topology, let us assume V-plus, we want to reach 85% within 400 meters. From [5], we know that the probability distribution of the individual distances of the houses to the active node can be estimated by a Normal distribution $F_{\mu,\sigma}(x)$ with $\mu_2 = \frac{D_2}{n_2}$ and $\sigma_2 = \frac{M-\mu}{2}$. Here M represents the maximum cable distance in the second topology using [6]:

$$M = 2 \cdot \lceil \frac{1}{2} s_2 - 1 \rceil \cdot d + 0.5d, \tag{70}$$

$$s_2 = \sqrt{n_2}, \tag{71}$$

and the total cable length in the second topology

$$D_2 = 2 \cdot d \cdot s_2 \cdot \lceil \frac{1}{2} s_2 \rceil \cdot \lfloor \frac{1}{2} s_2 \rfloor. \tag{72}$$

Now, the question is to choose n_2 such that $F_{\mu(n_2),\sigma(n_2)}(400) = 0.85$. This can be solved numerically and leads to the following values: $n_2 = 100$, $M_2 = 490$, $D_2 = 28800$, $\mu_2 = 290$ and $\sigma_2 = 100$. This means that to meet this requirement of 85% within 400 meter, 10 new nodes (n_1/n_2) should be installed. It takes 28800 meter of digging and (fibre) cable to connect these nodes.

VI. Solution Methods

The time to solve the Migration problem has to be of a reasonable magnitude, regardless of the input of the model. The reason for this is that the telecom operators should be able to run the optimisation model in a few minutes, such that the model can be used in an interactive way. After obtaining a migration plan, the company has to consider whether the migration plan is enforceable. If it is not a feasible plan, they should be able to modify input or requirements and create a new migration plan. Furthermore, in Section IV, it is shown that the Migration of Fibre problem is NP-hard. For these two reasons, heuristic methods are developed to obtain a good solution within an acceptable computation time. A heuristic method is a procedure that is likely to discover a good and feasible solution, but not necessarily an optimal solution. In this chapter, we present the different heuristic solution methods used in this research. The third solution method which is developed, is the optimisation of the base problem and the extended problem per year. Next to these heuristic approaches, the exact solution method is used to create benchmark values.

A. Problem-based heuristic

The first method we used to obtain an good solution in a reasonable computation time, is a heuristic method which is based on the characteristics of the Migration of Fibre problem. The main characteristics of the base problem is the requested bandwidth percentage and the two main characteristics of the extended problem are the budget and the installation capacities. The heuristic starts with a solution in which the technology and topology combination in each year is equal to the start year, i.e., the current situation. The heuristic starts at the first year that has to be upgraded and upgrades the locations with the largest profit. When enough locations are upgraded to meet the constraints for that year, the heuristic continues with the same procedure for the next years. After this, a feasible solution is constructed. In this way, the quality of the solution is guaranteed. Next, we explain how this is implemented for the base and extended problems.

For the implementation of the problem-based heuristic, we distinguish the base problem and the extended problem. For both the problems a total profit matrix is made. For the base problem, the profit is based on the following ratio:

$$\frac{R_{jl}}{C_{ijl}}, \tag{73}$$

where R_{jl} is the matrix containing the mean values over all the distances $d \in D$. For the extended problem, the profit is based on the following ratio:

$$\frac{R_{jl}}{\frac{C_{ijl}}{B_t} + \frac{H_{ijl}}{IC_t}}. \tag{74}$$

By dividing C_{ijl} and H_{ijl} respectively by B_t and IC_t, the influence of the migration costs and required installation capacities are equivalent. Moreover, the profit matrix shows for each possible upgrade per location what the corresponding profit ratio is per year. After this matrix is made, the following steps are performed:

1) Construct a migration schedule in which the technology and topology combinations in each time period are equal to the start time period, i.e., there are no migration upgrades in this schedule.
2) Select the lowest time period $t \in T$ which has not been upgraded yet and which has to be upgraded (base model: requested bandwidth constraint) or which could be upgraded (extended model: there is budget and installation capacity left over).
3) Using the total profit matrix, a profit matrix is made for the current situation. This is a matrix containing

the profits for the selected time period $t \in T$ and the technology and topology combination $i \in I$ of the previous time period $(t-1) \in T$.

4) The upgrade with the highest ratio in the matrix, made in the previous step, is selected and is carried out in the migration schedule. Also the subsequent time periods of this location get the same upgrade.

5a. (Base model) Repeat step 4. until the migration schedule for the selected time period meets the required bandwidth constraint. For the base problem, this is the bandwidth constraint, and in this way, the migration schedule up to the selected time period has became a feasible schedule.

5b. (Extended model) Repeat step 4. until as much locations as possible are upgraded and the solution still meets the budget and installation capacity constraint. In this way, the migration schedule is still a feasible schedule.

6a (Base model) Repeat step 2 until 5, until every time period $t \in T$ is upgraded as much as needed, and then, the migration schedule feasible.

6b (Extended model) Repeat step 2 until 5, until every time period $t \in T$ is upgraded as much as possible, without losing feasibility.

Note that the two last steps of the problem based heuristic are dependent of the type of the model, i.e., the base or extended model. Next to the model-based heuristic, we have also used a meta-heuristic, which is described in the next section.

B. Simulated Annealing

The meta-heuristic used in this research is Simulated Annealing (SA). A meta-heuristic is a general solution method that provides general structures and strategy guidelines for developing a specific heuristic method. SA is a stochastic algorithm which searches for a global optimum and avoids getting stuck in local, non-global optima [22]. It is based on a heating and cooling process and simulates the energy changes in a system subjected to a cooling process until it converges to an equilibrium state.

From an initial solution s_0, the SA algorithm generates a random neighbour during each iteration. A neighbour is a (feasible) solution obtained by performing an operation on the current solution. If this neighbour is a better solution than the current solution, related to the corresponding values of the objective function, the neighbour solution will be accepted and becomes the new current solution. If this is not the case, the neighbour will be accepted with a certain probability, which depends on the current temperature. This probability is the Boltzmann probability:

$$P(\text{acceptance}) = e^{-\frac{|f(s')-f(s)|}{T}}, \tag{75}$$

where $|f(s') - f(s)|$ denotes the difference ΔE between the objective value of the generated neighbour s' and the objective value of the current state s. T denotes the temperature. During each M_{max} iterations of the algorithm, the temperature T decreases, whereby the probability of acceptance also decreases. The probability of acceptance also depends on the quality of the neighbour solution, i.e., the worse the neighbour solution, the lower the chance of acceptance. A cooling schedule $g(T)$ defines for each step r of the algorithm the temperature T_r. Due to the possibility of accepting worse solutions, the algorithm can escape an inferior local minimum. The algorithm stops after a predefined amount of iterations N_{max}. The overview in Algorithm 1 summarises the used steps based on [23].

Algorithm 1: Simulated Annealing algorithm

```
Input: Cooling schedule g(T) and data
s = s_0;
(initial solution)
T = T_max;
(starting temperature)
N = 0;
while N < N_max do
    M = 0;
    while M < M_max do
        Generate a random neighbour s';
        ΔE = f(s') − f(s);
        if ΔE ≤ 0 for minimisation problem or ΔE ≥ 0
         for maximisation problem then
            s = s';
            (accept the neighbour solution);
        else
            Accept s' with probability e^(−|ΔE|/T) ;
            s = s' if s' is accepted ;
        end
        save s' and f(s') if s' is accepted;
        M = M + 1;
        N = N + 1
    end
    T = g(T);
end
Output: Saved solutions s' and corresponding objective
        values f(s')
```

The first step of our implementation of SA is to gain a good initial solution. To create an initial solution, the Problem-Based Heuristic as described in Section VI-A is used. Solutions are presented as a matrix of which the rows illustrate the locations, the columns represent the migrations years and the elements of the matrix represent the technology and topology combination for the corresponding location and year. The technology and topology combinations are ranged from 1 until k, with k the total amount of combinations. Furthermore, combination 1 provides the smallest bandwidth and combination k provides the largest bandwidth.

The objective function for a solution s of the base model is described as:

$$f(s) = \sum_{i \in I} \sum_{j \in I} \sum_{l \in L} \sum_{t \in T} C_{ijl} x_{ijlt}. \tag{76}$$

The objective function for a solution s of the extended model is described as:

$$f(s) = \sum_{t \in T} \sum_{d \in D} \max \left(0, G_{td} - \frac{\sum\limits_{i \in I} \sum\limits_{l \in L} R_{ild} \cdot y_{ilt}}{\sum\limits_{l \in L} RT_l} \right). \tag{77}$$

The goal of Simulated Annealing is to find a solution with the lowest possible objective value. Simulated Annealing also needs a temperature scheme. This scheme defines for each step of the algorithm the temperature T. First, we set an initial temperature T and we define the cooling schedule as:

$$g(T) = \alpha T \text{ , with } 0 < \alpha < 1. \tag{78}$$

We apply this scheme after each β^{th} iteration. Previous research showed that α should be between 0.5 and 0.99 [23]. The stop condition is defined as that the algorithm will stop after γ amount of iterations. In each iteration of the algorithm, a neighbourhood solution will be created, using the current solution. There are three operations possible to create a feasible neighbour solution. First, choose a random number. If the selected number is smaller than $\frac{1}{3}$, then operation 1 is performed, if the number is smaller than $\frac{2}{3}$ and bigger than $\frac{1}{3}$, then operation 2 is performed and otherwise, operation 3 is performed. By operation 1, a location is upgraded in a time period and, if possible, an other location is downgraded in the same time period. By operation 2, a location will be upgraded in a timed period and by operation 3, a location will be downgraded in a time period. The operations are specified as follows:

1) A location is randomly chosen. If the selected location contains already the best possible technology and topology combination in each migration time period, reselect the location randomly, until upgrading in at least one of the migration time periods of this location is possible. Then select a migration time period randomly, where upgrading the technology and topology combination for this location is possible and upgrade the selected location for the selected time period. With upgrading a network, we mean that we add 1 to the corresponding entry in the solution matrix. If needed, some of the following time periods for this location should also be increased by 1, such that the migration steps for the location form a row of non-descending entries.
   If it is possible to downgrade an other location in the selected time period, select randomly an other location and check if it is possible to downgrade this location in the selected time period. With downgrading a network, we mean that we subtract 1 from the corresponding entry. If the technology and topology combination for this location and time period is already as low as possible, then reselect the location randomly. This is repeated until a location is found where a downgrade is still possible in the selected time period and then the location in this time period is downgraded. In addition, if needed, some of the previous time periods for this location should be decreased by 1, such that the migration steps for the location form a row of non-descending entries. If it is not possible to downgrade an other location in the selected time period, no additional steps are performed.
2) A location is randomly chosen. If the selected location contains already the best possible technology and topology combination in each migration time period, reselect the location randomly, until upgrading in at least one of the migration time periods of this location is possible. Then, select a migration time period randomly, where upgrading the technology and topology combination for this location is possible and upgrade the selected location for the selected time period. With upgrading a network, we mean that we add 1 to the corresponding entry in the solution matrix. If needed, some of the following time periods for this location should also be increased by 1, such that the migration steps for the location form a row of non-descending entries.
3) A location is randomly chosen. If the selected location contains already the worst possible technology and topology combination in each migration time period, reselect the location randomly, until downgrading in at least one of the migration time periods of this location is possible. Then, select a migration time period randomly, where downgrading the technology and topology combination for this location is possible and downgrade the selected location for the selected time period. With downgrading a network, we mean that we subtract 1 from the corresponding entry in the solution matrix. If needed, some of the previous time periods for this location should be decreased by 1, such that the migration steps for the location form a row of non-descending entries.

For the extended problem, we added a small extension to operation 2 and 3, to increase the chance of creating a solution which is feasible:

2. Check the feasibility of the adapted solution. If it is infeasible, i.e., it does not meet the budget and/or installation capacity constraint, then also perform operation 3 in the selected time period. In this case, a location is upgraded and an other location is downgraded in the same time period.
3. Check the feasibility of the adapted solution. After operation 3 is performed, i.e., a location is downgraded in a time period, it is possible that the costs for the next time period becomes higher and exceeds the budget for this next time period. If the created neighbour solution is infeasible, i.e., it does not meet the budget and/or installation capacity constraint, then also perform operation 2 in the selected time period. In this case, a location is upgraded and an other location is downgraded

in the same time period.

For the base problem, check if the new solution is feasible, i.e., it meets the bandwidth constraint. For the extended problem, if an extension of operation 2 or 3 is performed, also a check has to be performed: it is checked whether or not the new solution meets the budget and installation capacity constraints. If it does not meet these constraints, the adapted solution is rejected and the procedure of the operation is started again, using the unadapted solution. This is repeated until a solution is found which meets the constraints. We call this adapted solution a neighbour. All the solutions which can be formed by using one of the operations to adapt the current solution, form the neighbourhood of the current solution. Additionally, during each iteration, the neighbour will be compared with the current solution. It will be accepted if it is better than the current solution and otherwise it will be accepted with the Boltzmann probability.

VII. Computational Results

We tested all methods described in the previous section, using the base model and extended model for various test instances. In this section, the results of these methods are presented. First of all, the impact of optimising per year instead of optimising over the total horizon is showed in an example. Next, the heuristics described in section VI are compared to each other, subjected to the accuracy and the computation time of these heuristics for various test instances.

A. Example

First, in this section, an example is presented introducing a small city with 40 cabinets and 18,550 houses, to show the benefit of the optimisation over the total horizon. The current employment is ADSL. The operator has a bandwidth coverage goal, expressed in percentage of the houses that is within a certain distance from the active equipment. The coverage goal is shown in Table III. For example, the goal is to have 70% of the houses within 400 meter in 2021.

Table III. Coverage goal.

Year	600m	400m	200m
2018	70%	40%	20%
2021	85%	70%	30%
2024	85%	85%	40%

Table IV. Per period optimisation - base model.

Year	ADSL	VDSL	V-plus
2018	23	7	10
2021	17	7	16
2024	5	7	28

Table V. Overall optimisation - base model.

Year	ADSL	VDSL	V-plus
2018	25	0	15
2021	18	1	21
2024	8	6	26

Two cases are distinguished. In the first case, the operator tries to meet the distance requirement for each year independently and optimally. This means that the operator optimises the design of each area without knowledge of future networks, topologies and technologies. In the second case, the operator tries to meet the requirements for the total time horizon, by solving the ILP model introduced in Section III. For each cabinet, for each 3-year period, the operator can chose between doing nothing, implementing VDSL and implementing V-plus, each with its own costs and bandwidth consequences. Now, the operator tries to make the decisions such that the total migration costs are minimal, meeting the distance coverage requirements for each period as modelled in the base model of Section III. The used costs for digging and equipment are based on the (Sub-Urban) numbers of [24].

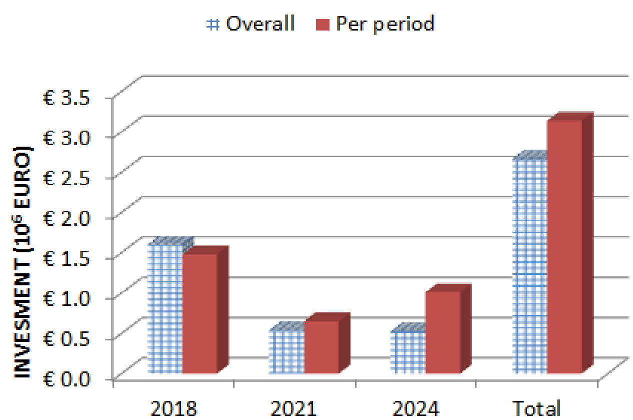

Figure 3. Investment costs.

The result of the optimisation (only using Excel and Open-Solver [25]) of the two cases is depicted in Table IV for the per-period optimisation and Table V for the overall optimisation. In the first year (2018) of the per-period optimisation, more VDSL is chosen, as this is a cheaper solution to meet the 2018 requirements. In the overall optimisation the more expensive choice for V-plus is made, as this is more ready for the future. In the other two stages more or less the same choices are made. This leads to the total overview of costs as depicted in Fig. 3, where the total costs of the overall optimisation are lower, but the costs in the first year are higher. All costs are expressed in Net Present Value, with an average cost of capital of 6%, making the values in the various years comparable.

B. Scalability

To illustrate the scalability of the problem solving, five real life areas containing different amounts of cabinets and houses are selected. The time-span of the migration schedule and the amount of possible technology and topology combinations is the same as in the previous example, respectively three moments in time and three combinations. In Table VI, an overview is given of the characteristics of the areas.

TABLE VI. AREA CHARACTERISTICS

Area	No. of cabinets	No. of houses
A	40	18,550
B	180	58,842
C	496	44,151
D	874	433,092
E	26164	6,352,365

The scalability of the base model and extended model is based on the quality of the optimal solution found by the different solutions methods and the corresponding computation times of these methods. To determine the quality of the provided solution of the methods, first the exact solutions of the base model and extended model for the different areas are calculated. For this, three different solvers are used. The first used solver is 'OpenSolver' in Excel, in combination with COIN-OR [25]. Furthermore, CPLEX Optimizer [26] in combination with MATLAB and the standard solver IntLinProg in MATLAB are used. In Table VII, an overview is given of the computation times using these solvers for the base problem.

TABLE VII. COMPUTATION TIME OF EXACT SOLUTION METHOD FOR THE BASE PROBLEM [SECONDS]

Area	COIN-OR	CPLEX Optimizer	IntLinProg
A	7.5	6.7	110.8
B	36.1	8.3	$>$ 8 days
C	396.3	9.7	$>$ 8 days
D	912.3	15.7	$>$ 8 days
E	$>$ 8 days	12,082.6	$>$ 8 days

TABLE VIII. COMPUTATION TIME OF EXACT SOLUTION METHOD FOR THE EXTENDED PROBLEM [SECONDS]

Area	CPLEX Optimizer
A	28.2
B	17.9
C	37.1
D	111.6
E	24,703.2

The illustrated computation times of the solvers are a combination of the time for building the problem and solving the problem using these solvers. Observe that the NP-hardness of the base model, effects the computation time for area E, using CPLEX Optimizer. This computation time is larger than three hours, which is not an reasonable duration for the telecom operators to obtain a migration schedule. In practise, even more technology and combinations and more migrations periods will be involved, which results in an exponential growing runtime for obtaining an exact solution. In Table VIII, an overview is given of the computation times for extended model using CPLEX Optimizer. This table also shows that the computation time for obtaining a migration schedule for large areas is not acceptable, given the assumptions in the start of this section.

Therefore, to obtain a good solution in a reasonable time, solution methods were developed, as shown in Section VI. These methods are the Problem-based heuristic and Simulated Annealing approaches and, additionally, a per period optimisation, calculating the optimal exact solution per year, sequentially. Simulated Annealing uses the solution of the problem-based heuristic as start solution. The parameters used to simulate the cooling process, based on preliminary results, are illustrated in Table IX for the base model and the extended model.

TABLE IX. BEST PARAMETERS FOR SIMULATED ANNEALING

Area	M_{max}	N_{max}	T_{max}	α
Base model	50	100,000	5,000	0.99
Extended model	50	20,000	0.01	0.95

Preliminary results show that the size of the areas has no effect on the selection of parameters, meaning that the best parameters is only based on the type of model. Now, the solutions of the three methods can be compared with the optimal solution found by CPLEX Optimizer. This is illustrated in Table X for the base model and in Table XI for the extended model.

TABLE X. SOLUTION OF THE HEURISTICS COMPARED TO THE EXACT SOLUTION OF BASE MODEL (COSTS)

Area	Per period opt.	Problem-based	Simulated Annealing
A	23.0%	6.1%	0.9%
B	0.0%	29.9%	12.4%
C	2.4%	30.5%	23.3%
D	0.0%	22.4%	18.5%
E	11.5%	12.1%	10.9%

TABLE XI. SOLUTION OF THE HEURISTICS COMPARED TO THE EXACT SOLUTION OF EXTENDED MODEL (BANDWIDTH)

Area	Per period opt.	Problem-based	Simulated Annealing
A	99.74%	54.57%	98.12%
B	100.00%	83.65%	99.40%
C	99.86%	85.25%	99.69%
D	100.00%	89.89%	99.38%
E	99.94%	76.05%	98.01%

Table X shows that the solution for the base problem in area A, using Simulated Annealing, is 2.6% worse than the exact optimal solution, as found by CPLEX Optimizer. This means that the costs for the migration schedule as found by Simulated Annealing is 2.6% higher than the costs for the migration schedule of the exact solution, as found by CPLEX Optimizer. Table XI shows that the solution for the extended problem in area A, using the problem-based heuristic, has a total realisation of 54.57% of the total realised bandwidth demand of the exact optimal solution, as found by CPLEX Optimizer. Moreover, the 100% of the exact solution is equal to the sum over the minima of the demanded bandwidth and the realised bandwidth per year and distance. Furthermore, these two tables show that the improvement using Simulated Annealing is significantly more effective for the extended model. However, the optimal solutions of Simulated Annealing for both the models are not as good as the optimal solutions of the per period optimisation.

The corresponding computation times of the methods are illustrated in Table XII and Table XIII. Note that the computation time of Simulated Annealing includes the computation time of the problem-based heuristic.

TABLE XII. COMPUTATION TIMES OF THE HEURISTICS FOR THE BASE MODEL [SECONDS]

Area	Per period opt.	Problem-based	Simulated Annealing
A	17.5	8.0	16.6
B	16.3	7.9	40.9
C	19.6	7.7	126.4
D	21.2	9.0	226.3
E	1181.3	694.3	1,242.4

TABLE XIII. COMPUTATION TIMES OF THE HEURISTICS FOR THE EXTENDED MODEL [SECONDS]

Area	Per period opt.	Problem-based	Simulated Annealing
A	26.8	13.1	20.1
B	23.6	10.5	29.2
C	26.8	14.2	39.3
D	29.6	7.2	164.8
E	1,045.7	626.4	1,783.2

Table XII and Table XIII show that the computation time of the problem-based heuristic is the lowest, however, the computation time of the two other methods are also of a reasonable duration. To conclude, combining this with the previous results that the solution of the per year optimisation is significantly better than the solution provided by the problem-based heuristic, the best method to provide a good solution in a reasonable time for the extended model, is the approach of optimising per year. For the base model, this approach can give rather high deviations. Then, for smaller areas, at this moment, the exact solver should be considered.

VIII. CONCLUSION

In this paper, we presented a methodology that can be used by operators to design their heterogeneous topology migration path from Full Copper to (Full) FttH, meeting their business requirements. Heterogeneous means that the operator decides per Central Office area the topology or technique per period (e.g., year), resulting in a detailed migration path that meets a required bandwidth coverage in the larger area. For this, two models were presented. The first, the base problem, minimised the total investment (CapEx) and operational costs (OpEx), such that the bandwidth requirement per period was met. The second, the extended problem, minimised the deviation from this bandwidth requirement meeting a budget constraint per period.

The data used for the migration path optimisation is in practice hard to obtain. For this, the use of geometric modelling was proposed, with which the total CapEx of a migration step can be estimated using only two parameters per Central Office area, namely the total existing cable length and the capacity of this node. The two models were demonstrated in two case studies that showed the gain that can be realised by the migration path optimisation.

To be used in practice, solving bigger instances, two heuristic methods were presented, next to the option to solve the problem per year. Numerical experiments show that an exact optimal solution can be obtained by MIP-solver CPLEX Optimizer up to rather big problems. If for bigger problems the calculation time of the exact solution is experienced to high, a consecutive approach of optimising per year gives the best performance of the heuristic approaches, in the case of the extended problem.

For further research we recommend to look for better performing problem-based heuristics, as we expect them to deliver the best computation times. Furthermore, beside the costs, also the revenues could be taken into account.

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