Modeling Systems with Multi-service Overflow Erlang and Engset Traffic Streams

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Abstract

The article proposes analytical methods for determining traffic characteristics of hierarchically organised telecommunication networks which are offered multi-service traffic streams. The article proposes a method for determining occupancy distribution in the group servicing multi-service overflow traffic. This method is based on modification of the Kaufman-Roberts recursion – elaborated for the full-availability group with Poisson calls streams – and uses Fredericks & Hayward approximation. Additionally, a method for determining parameters of the traffic overflowing from primary groups servicing PCT1[1] and PCT2[2] traffic streams is also presented.
Keywords: *overflow traffic, PCT1, PCT2, multi-rate traffic*

1. Introduction

Modeling telecommunication networks employing the strategy of redirecting traffic via alternative routes, i.e. systems with traffic overflow is a complex issue. This problem comes down to resolving the two following basic problems, namely: to a determination of traffic characteristics of traffic that overflows from direct (primary) groups (with high loss coefficients usually), and a determination of the number the so-called Basic Bandwidth Units (or channels) in alternative groups (with low loss coefficients usually), where the loss coefficients will not exceed the assigned value.

Systems with overflow traffic have been widely discussed e.g., in [8,22,35]. The above mentioned works, however, have dealt with single-rate traffic only, i.e. with traditional single-service telephone networks. There have been developed both exact [4,14,25,36] and approximate [15,35] models of the full-availability group with overflow traffic assuming Poisson distribution of calls streams and the exponential distribution of holding time for calls offered to the primary groups. The problem of modeling the groups with overflow traffic under assumption of hyper-exponential distribution of the holding time has been described in [27] while single-rate traffic systems with overflow traffic and finite number of traffic sources (PCT2) have been considered e.g., in [26].

The basic method for determining traffic characteristics of multi-service systems employs the so-called Kaufman-Roberts formulas (KR) [19, 24]. These equations allow to reliably model systems with PCT1 streams that are offered directly to the primary groups of telecommunication networks. The traffic that is not serviced in such groups is overflowed to an alternative group. This part of traffic is called the overflow traffic. However, even if the streams that are offered directly to the primary groups are of type PCT1, the calls stream overflowing from the primary group does not agree with the Poison distribution [35].

Overflow calls can appear only in the occupancy time of all Basic Bandwidth Units of the primary group. This means that the overflow stream is more "concentrated" in certain time periods, i.e. is characterized by greater "peakedness" as compared with PCT1 traffic. If identical values of offered traffic and the congestion are assumed, then a greater number of Basic Bandwidth Units (BBUs) is required for servicing overflow traffic than that required for servicing PCT1 traffic.

The following parameters can be used for statistical evaluation of the overflow stream: the mean value R of overflow traffic (the first moment of the probability distribution of the number of calls) and the second moment with the corresponding variance σ^2. With the help of those two parameters it is possible to determine "unevenness" of the overflow stream by the introduction of the concept of the peakedness

[1]PCT1 – Pure Chance Traffic Type One – type of traffic in which we assume that the service times are exponentially distributed and the arrival process is a Poisson process. This type of traffic is known as Erlang traffic.

[2]PCT2 – Pure Chance Traffic type Two – type of traffic in which we assume that the service times are exponentially distributed and the arrival process is formed by the limited number of sources. This type of traffic is known as Engset traffic.

coefficient Z that is equal to the ratio of the variance σ^2 to the mean value of overflow traffic R:

$$Z = \sigma^2/R. \tag{1}$$

The "unevenness" of the overflow stream can also be evaluated by the application of the parameter D that is the difference between the variance and the mean value of overflow traffic:

$$D = \sigma^2 - R. \tag{2}$$

It is noticeable that the parameters Z and D take the following values for the offered traffic, serviced traffic and the overflowed traffic:

- for offered traffic: $Z = 1$ and $D = 0$,
- for serviced traffic on the primary group (smooth traffic): $Z < 1$ and $D < 0$,
- for overflow traffic: $Z > 1$ and $D > 0$.

The service process of a Poisson calls stream in a full-availability group can be thus characterized by four parameters A, V, R, σ^2 (σ^2 can be replaced by Z or D). The stream offered to the group is here determined by one parameter A – the mean value of the offered traffic, whereas the overflow traffic stream by two: the mean value of the overflow traffic R and its variance σ^2.

Having the above in mind, we can come to a conclusion that the KR equations in their basic form (devised with the assumption of the exponential distribution of time gaps between calls) cannot be applied to determining call blocking coefficients in multi-service traffic in the alternative group. The problem of modeling the full-availability group with overflow traffic with known value of parameter Z was taken in [7], and then in [20, 34]. The methods for modeling the systems with multi-service overflow traffic (under the assumption of infinite number of traffic sources) including the methods for determining parameters of overflow traffic, an occupancy distribution in alternative groups and dimensioning systems with multi-service overflow traffic was presented in [10, 11, 13].

The other group of methods, enabling modeling the systems with overflow traffic, are the methods based on Markov-Modulated Poisson Processes, published in [6, 17, 21]. Among this group of methods, the highest accuracy, in case of multi-service systems, assures the method proposed in [6]. The accuracy of this method is related to high computational complexity of the process of calculating the variance of overflow traffic based on analysis of multidimensional Markov process in the system composed of two groups, i.e. the primary group and the alternative group. Exponential order of computational complexity (in function of number of classes of calls) makes practical application of this method very difficult.

The purpose of the article is the proposition of a consistent methodology for determining traffic characteristics of systems which are offered overflow multi-service traffic streams, generated both by finite and infinite source population. On the basis of author's earliest results [10–13]), the method for determining occupancy distribution in the group servicing multi-service overflow traffic will be presented. The proposed method is based on the appropriate modification of the Kaufman-Roberts recursion [19, 24] – elaborated for the full-availability group with Poisson traffic – and uses the idea of Fredericks & Hayward approximation.

In order to keep consistency of the considered problems, we start considerations from presentation of basic analytical dependencies for systems with single-rate overflow traffic in Section 2. In Section 3 it is presented the method for determining occupancy distribution in groups servicing multi-service overflow traffic. Section 4 includes the description of the method for determining parameters of the traffic overflowing from primary groups servicing multi-service PCT1 and PCT2 traffic streams. Comparison of analytical and simulation results of blocking probability in alternative groups servicing multi-service overflow traffic is performed in Section 5. Section 6 concludes the paper.

2. Modeling systems with overflow single-rate traffic

2.1. Overflow traffic parameters

The traffic that overflows from the direct group which is offered PCT1 traffic can be characterized with the help of the following two parameters: the mean value of overflow traffic R and its variance σ^2 (or the coefficient Z or the coefficient D). In order to evaluate analytically these parameters we will consider the following model: a full-availability group with the capacity of V Basic Bandwidth Units (the primary group) is offered traffic of the type PCT1 with the mean intensity A:

$$A = \frac{\lambda}{\mu}. \tag{3}$$

The next assumption is that the traffic that is not carried because of the occupancy of all the BBUs of the considered group overflows to a next full-availability group (the alternative group) with an unlimited number of BBUs. The values to be determined are: the average number of busy BBUs R in the alternative group (mean value of overflow traffic) and its variance σ^2 (variance of overflow traffic).

The process going on in the system presented in Figure 1, composed of two full-availability groups, is determined by the the two-dimensional discrete Markov chain: $\{\omega(t), \rho(t)\}$, where $\omega(t)$ is the number of busy BBUs in

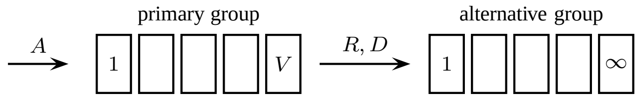

Figure 1. Model of a system with overflow traffic

the original group at the point of time t, whereas $\rho(t)$ is the number of busy BBUs in the alternative group at the point of time t. The state probabilities of the system under consideration are denoted with the symbols $[p_{\omega,\rho}]_{V,\infty}$ and are defined in the following way:

$$[p_{\omega,\rho}]_{V,\infty} = \lim_{t\to\infty} P\left\{\omega(t)=\omega, \rho(t)=\rho\right\}, \qquad (4)$$

where: $(0 \leqslant \omega \ \leqslant V)$ and $(0 \leqslant \rho \leqslant \infty)$. The probabilities $[p_{\omega,\rho}]_{V,\infty}$ can be determined on the basis of the system of state equations that, for the considered process, takes the following form:

$$\begin{gathered}
\ldots \\
-(\lambda+\rho\mu)\left[p_{0,\rho}\right]_{V,\infty} + \mu\left[p_{1,\rho}\right]_{V,\infty} + \\
+(\rho+1)\mu\left[p_{0,\rho+1}\right]_{V,\infty} = 0 \ldots \\
-(\lambda+\omega\mu+\rho\mu)\left[p_{\omega,\rho}\right]_{V,\infty} + \lambda\left[p_{\omega-1,\rho}\right]_{V,\infty} + \\
+(\omega+1)\mu\left[p_{\omega+1,\rho}\right]_{V,\infty} + (\rho+1)\mu\left[p_{\omega,\rho+1}\right]_{V,\infty} = 0 \\
\ldots \qquad (5) \\
-(\lambda+V\mu+\rho\mu)\left[p_{V,\rho}\right]_{V,\infty} + \lambda\left[p_{V-1,\rho}\right]_{V,\infty} + \\
+\lambda\left[p_{V,\rho-1}\right]_{V,\infty} + (\rho+1)\mu\left[p_{V,\rho+1}\right]_{V,\infty} = 0 \\
\ldots \\
\sum_{\rho=0}^{\infty}\sum_{\omega=0}^{V}\left[p_{\omega,\rho}\right]_{V,\infty} = 1
\end{gathered}$$

Once the system of equations (5) has been solved, it is possible to determine all essential properties of the system with traffic overflow. A determination of the parameters R and σ^2, related to the alternative group with unlimited capacity, can be, however, simplified as compared to the system (5). This possibility of simplification is connected with the fact that for a determination of parameters R and σ^2 the knowledge of all probabilities $[g_\rho]_\infty$ is not necessary, but it is sufficient to know only those probabilities $[g_\rho]_\infty$ that relate to the alternative group only, regardless the occupancy state of the primary group, i.e.:

$$[g_\rho]_\infty = \sum_{\omega=0}^{V} [p_{\omega,\rho}]_{V,\infty}. \qquad (6)$$

Knowing the occupancy $[g_\rho]_\infty$, it is possible to determine the parameters to be found, i.e. R and σ^2:

$$R = \sum_{\rho=0}^{V} \rho\left[g_\rho\right]_\infty, \quad \sigma^2 = \sum_{\rho=0}^{V} \rho^2\left[g_\rho\right]_\infty - R^2. \qquad (7)$$

Derivations of Equation (7) will be omitted here (they are to be found in, for example, [1, 4, 35]), by giving the final result derived by J. Riordan [35]:

$$R = AE_V(A), \qquad (8)$$

$$\sigma^2 = R\left[A/\left(V+1-A+R\right)+1-R\right]. \qquad (9)$$

In calculational practice, instead of the variance σ^2 the parameter D is often used. Hence, on the basis of Equation (2), (8) and (9) we obtain:

$$D = R\left[A/\left(V+1-A+R\right)-R\right]. \qquad (10)$$

Formula (8) is intuitively self-evident since it is only traffic lost in the original group that can be the offered traffic and, at the same time, be carried by the infinite alternative group. It should be noted that, quite predictably, for $V=0$ (zero capacity of the original group), $R=\sigma^2=A$, since all the PCT1 traffic is directed to the alternative group. Generally, for each value of the parameters A and V of the full-availability group, the parameters of overflow traffic R and σ^2, or R and D can be unequivocally determined.

In telecommunications networks, calls streams from several high-usage full-availability groups most frequently overflow to one alternative path. If we assume that PCT1 streams offered to high-usage primary groups are statistically independent, then the streams that overflow from these groups will also be independent. In such a case, the parameters of the total overflow traffic offered to the alternative path are determined by the following formulas [31]:

$$R = \sum_{s=0}^{\upsilon} R_s, \quad \sigma^2 = \sum_{s=0}^{\upsilon} \sigma_s^2, \quad D = \sum_{s=0}^{\upsilon} D_s, \qquad (11)$$

where: υ – number of primary group, R_s – mean value of overflow traffic from s-th group, σ_s^2 – variance of overflow traffic from s-th group.

2.2. Method of equivalent random traffic

Analysing Formulas (8) and (10) we can notice that the parameters A and V determine unequivocally the parameters of the overflow traffic R and D of a given group. Consequently, these formulas can be used to solve a reverse problem, i.e. to determine unequivocally the parameters of the original group A and V on the basis of the parameters of the traffic that overflows from this group: R and D [31]. This conclusion has been applied to the ERT method (Equivalent

Random Traffic), which has been worked out independently by R. I. Wilkinson [35] and G. Bretschneider [4].

The ERT method consists in finding such an equivalent PCT1 traffic with the mean value A^*, that when offered to a fictitious equivalent group with the equivalent capacity of V^*, will cause an overflow of traffic with identical mean value and variance as the actual traffic offered to a given alternative group [31]. In this way, the traffic initially defined by the pairs of parameters: A_s and V_s (the alternative group usually services traffic overflowing from a few high-usage primary groups), will be described by one pair of parameters only (A^*, V^*).

The parameters A^* and V^* of the equivalent group can be determined on the basis of the obtained values R and D, solving the set of Riordan equations [35]:

$$R = A^* E_{V^*}(A^*), \tag{12}$$

$$D = R\left[A^* / \left(V^* + 1 - A^* + R\right) - R\right]. \tag{13}$$

Such equivalent traffic, determined by the pair of the parameters (A^*, V^*), requires $V^* + V_{\text{alt}}$ BBUs for servicing calls with assigned quality B. The required capacity of the alternative group can be obtained on the basis of Erlang-B formula, written in the following form:

$$E = B = E_{(V^* + V_{\text{alt}})}(A^*), \tag{14}$$

where E is the blocking probability, and B is the loss probability in the alternative group.

Summing up, the ERT method, presented graphically in Figure 2, can be written in the form of the following algorithm:

Algorithm 1 ERT Method

1. Determination of the mean value R_s and the parameter D_s of each of υ $(s = 1, \ldots, \upsilon)$ traffic streams that overflow to the alternative group (Equations (8) and (10));

2. Determination of the parameters of the total stream that overflows to the considered alternative group, assuming statistical independence of overflow streams (Equation (11));

3. Determination of the parameters A^* and V^* of the equivalent group on the basis of the obtained parameters R and D; these parameters can be determined by providing solution to the Riordan system of equations (Equation (13));

4. Determination of the required capacity of the alternative group for the assigned quality of service in the system equal to B (Equation (14)).

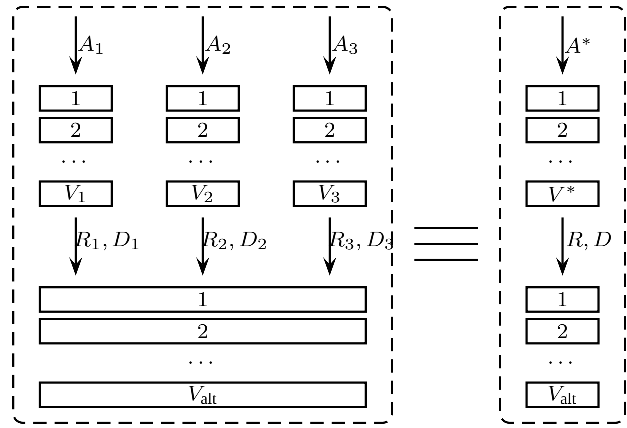

Figure 2. Graphical representation of the ERT method

The determination of the parameters of the equivalent group (A^*, V^*) is a complex issue and requires the application of complex, iterative computational programs [23, 31]. Therefore, to simplify the calculations, special nomograms have been developed [28] that present in graphic form dependencies between pairs of parameters (A^*, V^*) and (R, D). If, however, the above graphic dependencies are unavailable, then to determine the parameters (A^*, V^*) one can use the approximate solution of the system of equations (12) and (13), proposed by G. Rapp [22]:

$$A^* = \sigma^2 + 3\frac{\sigma^2}{R}\left(\frac{\sigma^2}{R} - 1\right), \tag{15}$$

$$V^* = A^* \frac{(R^2 + \sigma^2)}{R^2 + \sigma^2 - R} - R - 1. \tag{16}$$

It should be stressed that the determined values of parameters A^* and V^* obtained after the application of Rapp formulas are approximate, with the accuracy of calculations being the lowest within the area of low loss probability values [33]. With values of this probability lower than 1%, the approximation error can exceed 20%. Therefore, for $B < 0.01$ (which happens rarely in high-usage primary groups in real networks) it is more convenient to use the cited above nomograms [28]. A detailed analysis of the accuracy of this method has been worked out by J. M. Holtzmann and presented in [16] which shows the dependency between the error of loss probability, determined by the ERT method, and the number of BBUs of the alternative group V_{alt} and the overflow traffic parameters R and σ^2. On the basis of these dependencies it is possible to find that the error increases with the increase of the variance of overflow traffic σ^2, while it diminishes along with the increase in the number of BBUs in the high-usage primary group [31].

2.3. Fredericks-Hayward Method

Let us consider a full-availability group with the capacity of V BBUs which is offered overflow traffic with the mean value R and variance σ^2. The peakedness coefficient of the offered traffic is then:

$$Z = \frac{\sigma^2(R)}{R}. \tag{17}$$

Let us perform the following transformation, presented in Figure 3. Let us divide the group into Z identical full-availability groups (subsystems), each one with the capacity:

$$V_e = \frac{V}{Z}. \tag{18}$$

Each group is offered then traffic with the mean value:

$$R_e = \frac{R}{Z}. \tag{19}$$

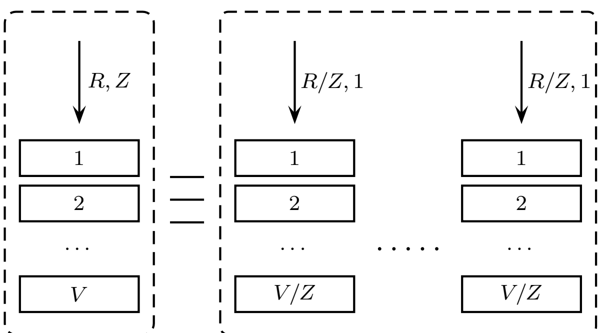

Figure 3. Transformation of the system (V, R, Z) into Z subsystems $(V/Z, R/Z, 1)$

Taking into consideration the property of variance, variance σ_e^2 can be determined in the following way:

$$\sigma_e^2 = \sigma^2\left(\frac{1}{Z}R\right) = \left(\frac{1}{Z}\right)^2 \sigma^2(R). \tag{20}$$

Now we can determine the peakedness coefficient of traffic offered to an individual subsystem. Taking into account (19) and (20), we get:

$$Z_e = \frac{\sigma_e^2}{R_e} = \frac{\sigma^2(R)}{RZ} = 1. \tag{21}$$

The peakedness coefficient equal to one means that traffic R_e is a PCT1 traffic. Thus, we have made a transformation of the full-availability group – described by the parameters (R, V, Z) – which is offered overflow traffic into Z subsystems (full-availability groups) – described by the parameters $(R/Z, V/Z, 1)$ – which is offered PCT1 traffic. Since all groups are identical, blocking probabilities in all groups will be also identical. In work [8] it is assumed that blocking probability in the group $(R/Z, V/Z, 1)$ will be the same as in the initial group (R, V, Z). Therefore, we can write:

$$E(R, V, Z) \approx E(R/Z, V/Z, 1) \approx E_{\frac{V}{Z}}\left(\frac{R}{Z}\right). \tag{22}$$

Formula (22) is a modified Erlang-B formula that takes into consideration non-Poisson nature of the calls stream offered to the group. In teletraffic theory, this formula is called Fredericks-Hayward formula.

The presented reasoning for Equation (22) assumes mutual independence of traffic offered to the subsystems. In real world, a distribution of the traffic stream into several identical streams without an application of an appropriate call assignment mechanism is not possible. The introduction of such a mechanism is, however, tantamount to the introduction of mutual correlation between the streams, which, in turn, can be interpreted as a lack of independence of the traffic streams offered to the subsystems. This phenomenon makes the formula (22) an approximated formula. It should be stressed, though that it is characterized by high accuracy [8, 18].

Equation (22) forms the basis for Fredericks-Hayward method [8] and can be described in the form of the following algorithm:

Algorithm 2 Fredericks-Hayward Algorithm

1. Determination of the mean value and the variance of each of υ traffic streams that overflows to an alternative group based on the formulas (8) and (9);
2. Determination of the parameters of the total overflow traffic (Equation (11)) offered to the alternative group and the peakedness coefficient (Equation (1)) of the traffic, assuming statistical independence of the overflow streams;
3. Determination of the number of BBUs of the alternative group (with the assigned quality of service, equal to B) on the basis of Fredericks-Hayward formula (22).

Fredericks-Hayward method is far more simple than the ERT method since it requires only calculations based on Erlang-B formula. The formula is used in two steps of the algorithm – with the determination of mean value of traffic that overflows to the alternative group (Formula (8)) and, in the form of Fredericks-Hayward formula, with the determination of the capacity of the alternative group (Formula (22)).

3. Modeling of full-availability groups with multi-service overflow traffic

3.1. Basic assumptions

Let us consider first a fragment of the network shown in Figure 4, servicing multi-service PCT1 traffic streams. It is assumed that each of primary groups is offered only one call class. The adopted assumption is to facilitate the understanding of the introduced analytical dependencies. Systems in which primary groups service many classes of traffic will be presented in Section 4.

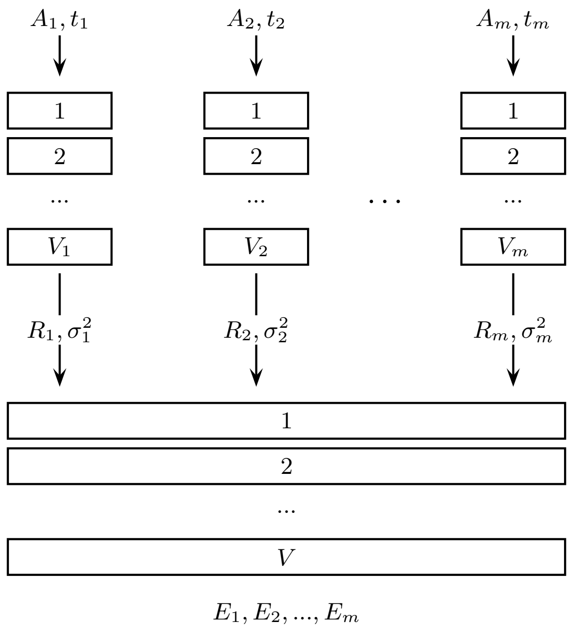

Figure 4. A fragment of the network with overflow traffic

There are $m = m_I$ high-usage primary groups in the considered system. The group designated by number i has the capacity equal to V_i BBUs. Each of the groups is offered a different calls stream characterized by the traffic intensity A_i. The calls of class i demand t_i BBUs to set up a connection.

3.2. Parameters of overflow traffic

As the result of occupying successive BBUs in primary groups, a situation ensues in which the groups get blocked and traffic overflows to an alternative group with the capacity V_{alt}. Blocking coefficients in primary groups can be calculated with the help of the Erlang-B formula. One has to take into consideration, however, that one call of class i occupies simultaneously t_i BBUs [10, 11].

Therefore, from the point of view of the Erlang model, it is tantamount to t_i-fold decrease of the capacity of the group with the real capacity of V_i BBUs. What it means is that before the substitution to Erlang-B formula, the group capacity should be divided by the number of BBUs demanded to set up a connection of a given class. With the case of non-integral values V_i/t_i, calculations of blocking probability can be performed using the interpolation method or the approximation of Erlang loss formula in the following form [32]:

$$E_{N+\delta} = \frac{AE_{N+\delta-1}(A)}{N + \delta + AE_{N+\delta-1}(A)}, \tag{23}$$

where $N + \delta$ is non-integral value of group's capacity (N is an integer part and δ is a fraction). To start the calculation process we need to use an approximate formula:

$$E_\delta \approx \frac{(2-\delta)A + A^2}{\delta + 2A + A^2}. \tag{24}$$

Another way to obtain the same values of blocking coefficients is to apply the Kaufman-Roberts formulas [19, 24]:

$$n\,[P_n]_V = \sum_{i=1}^{m} A_i t_i\,[P_{n-t_i}]_V, \tag{25}$$

$$B_i = E_i = \sum_{n=V-t_i+1}^{V} [P_n]_V, \tag{26}$$

where $[P_n]_V$ is the occupancy distribution, i.e. the probability of n BBUs being busy in the system. Equations (25) and (26) will take into consideration the group with the capacity of V_i which is offered one calls stream with Poisson distribution formed by the calls that demand t_i BBUs to set up a connection [10, 11].

Knowing the blocking coefficients in primary groups we are in position to calculate the parameters of overflow traffic of each of the classes, i.e the mean value R_i and the variance σ_i^2. For this purpose, the Riordan formulas (8) and (9) are used. Then, on the basis of the obtained parameters, we determine the unevenness of individual calls streams of overflow traffic by calculating the values of peakedness coefficients $Z_i = \sigma_i^2/R_i$.

It should be emphasised that the possibility of direct application of Riordan formulas, elaborated for systems with single-rate traffic, results from the assumption that each primary group is offered only one traffic class [10, 11]. In the case when all groups serve calls of several traffic classes, the determination of variance of overflow traffic becomes a complex problem [2, 3], despite the value of traffic intensity can be simply obtained on the Kaufman-Roberts formulas (25) and (26). An approximate method of elaboration of the variance of the traffic overflowing from primary group servicing mixture of multi-service traffic will be presented in Section 4.

3.3. Modeling overflow traffic in systems with infinite number of traffic sources

Calls lost in primary groups are offered to an alternative group and, successively, begin to occupy its resources. Thus, the group services m call classes. In order to determine blocking coefficients in such a group we apply the analogy to Hayword method, described in Section 2.3. Let us remind that the method was designed to determine the blocking coefficient in the group with the capacity of V BBUs with single-service traffic which was offered overflow traffic stream with the mean value R, additionally characterized by the peakedness Z. In this method the Fredericks-Hayword equation is used, i.e. the Erlang-B formula with appropriately modified parameters A and V. In the case of a group with multi-service traffic, we will apply the identical modification to Kaufman-Roberts formulas:

$$E_{\text{alt},1}, E_{\text{alt},2}, \ldots, E_{\text{alt},m} = \\ = KR\left(\frac{R_1}{Z_1}, \frac{R_2}{Z_2}, \ldots, \frac{R_m}{Z_m}; t_1, t_2, \ldots, t_m; \frac{V_{\text{alt}}}{Z}\right), \quad (27)$$

where $KR(\cdot)$ denotes the algorithm for determining blocking coefficients of calls of particular classes E_1, E_2, ..., E_M, on the basis of the Kaufman-Roberts equations (25) and (26) that take on the following form [10, 11]:

$$n\left[P_n\right]_{V_{\text{alt}}/Z} = \sum_{i=1}^{m} \frac{R_i}{Z_i} \cdot t_i \left[P_{n-t_i}\right]_{V_{\text{alt}}/Z}, \quad (28)$$

$$B_{\text{alt},i} = E_{\text{alt},i} = \sum_{n=\frac{V}{Z}-t_i+1}^{\frac{V_{\text{alt}}}{Z}} \left[P_n\right]_{V_{\text{alt}}/Z}. \quad (29)$$

The peakedness coefficient acts a normalization function. By dividing the mean values of overflow traffics of particular call classes by the corresponding values of the coefficients Z_i, we perform a transformation of the uneven overflow traffic stream into the Erlang stream. Similarly as in the dependence (22), we also divide the capacity of the alternative group V by the value of the peakedness coefficient. Let us notice that the capacity of the alternative group in the formulas (28) and (29) is divided by the so-called overall peakedness coefficient Z. The problem of definition of this coefficient, for m calls classes, where each can have individual value of the peakedness Z_i, was taken in [10]. According to these considerations, the relevant parameter will be approximated by the weighted mean of the coefficients Z_i of particular calls streams:

$$Z = \sum_{i=1}^{m} Z_i k_i, \quad (30)$$

where

$$k_i = \frac{R_i t_i}{\sum_{l=1}^{m} R_l t_l} \quad (31)$$

It is adopted in Equation (30) that the contribution of peakedness Z_i of a stream of class i in the overall peakedness coefficient Z is directly proportional to the value of traffic offered to the alternative group by class i calls. The plausibility of this assumption has been proved by simulation studies [13].

The formulas (28) and (29) are a generalization of the Kaufman-Roberts formulas for all kinds of groups servicing multi-service traffic, both non-Poisson calls streams (overflow traffic) and Poisson calls streams. For the Poisson distribution, the value of the peakedness is equal to one and then the formulas (28) and (29) will take on the form of the basic Kaufman-Roberts formulas (25) and (26).

3.4. Modeling of overflow traffic in systems with finite number of traffic sources

In this section it is presented an analytical method for determining the mean value and the variance in systems with multi-service traffic overflowing from primary groups servicing multi-service PCT2 traffic streams [12]. The presented method is based on the method elaborated in [5] for the networks servicing single-rate traffic. The basis of this method is the application of ERT method to convert the traffic stream generated by the finite population of sources (PCT2 traffic stream) to the equivalent traffic stream generated with the assumption of the infinite population of sources (PCT1 traffic streams) [29].

Let us consider a group with the capacity of V_j BBUs servicing a finite number of sources for each traffic class. Let N_j be a number of sources of class j requiring t_j BBUs to be serviced. The input calls stream of class j is built by the superposition of N_j two-state traffic sources which can alternate between the active (busy) state ON (the source requires t_j BBUs) and the inactive state OFF (the source is idle). When a source is busy, its call intensity is zero. Thus the arrival process is state-dependent. The class j arrival rate in the state of n BBUs being busy can be expressed by the following formula:

$$\lambda_j(n) = (N_j - n_j(n))\Lambda_j, \quad (32)$$

where $n_j(n)$ is a number of class j calls being serviced in state n (state of n BBUs being busy) and Λ_j is the mean arrival rate generated by an idle source of class j. In the considered model we assume additionally that the holding time for calls of particular classes has an exponential distribution. Thus, the class j traffic α_j offered by an idle source is equal to:

$$\alpha_i = \frac{\Lambda_i}{\mu_i}, \quad (33)$$

where $1/\mu_j$ is the mean holding (service) time of class j calls.

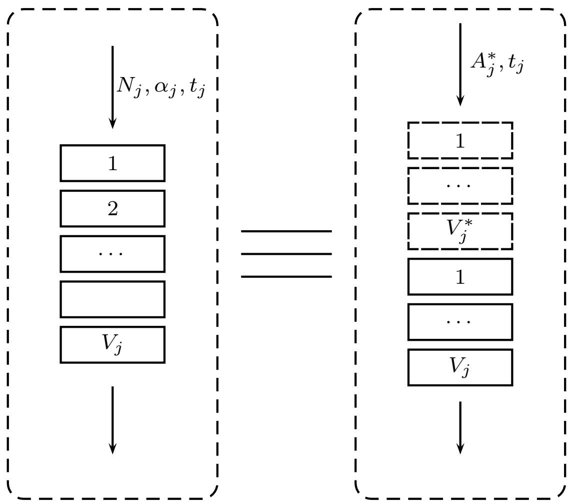

Figure 5. The idea of conversion of systems PCT2 to PCT1

Let us additionally assume, that $N_j > V_j$. Based on the results presented in [5] and [29] we can determine the mean value $R_{\mathrm{PCT2},j}$, the variance $\sigma^2_{\mathrm{PCT2},j}$ and the coefficient $D_{\mathrm{PCT2},j}$ of the number of busy BBUs in considered group:

$$R_{\mathrm{PCT2},j} = \frac{N_j \alpha_j}{1+\alpha_j}, \tag{34}$$

$$\sigma^2_{\mathrm{PCT2},j} = \frac{N_j \alpha_j}{(1+\alpha_j)^2}, \tag{35}$$

$$D_{\mathrm{PCT2},j} = \sigma^2_{\mathrm{PCT2},j} - R_{\mathrm{PCT2},j} = -N_j \frac{\alpha_j}{(1+\alpha_j)^2}. \tag{36}$$

The traffic described by Equations (34), (35) and (36) can be treated as an equivalent PCT1 stream with intensity A_j^* overflowing on the equivalent group with the capacity equal to V_j^* BBUs. The idea of this conversion is presented in Figure 5. We call A_j^* and V_j^* fictitious, and their values can be obtained as the solution of a set of Riordan formulas – according to ERT method (page 4):

$$R_{\mathrm{PCT2},j} = A_j^* E_{V_j^*}\left(A_j^*\right), \tag{37}$$

$$D_{\mathrm{PCT2},j} = \\ = R_{\mathrm{PCT2},j}\left[\frac{A_j^*}{V_i^* + 1 - A_j^* + R_{\mathrm{PCT2},j}} - R_{\mathrm{PCT2},j}\right]. \tag{38}$$

The above equations have a solution if we use Erlang formula for negative values of link capacity [5, 32]. It is possible to obtain the occupancy distribution for $V < 0$ on the basis of the following recurrent formula:

$$E_{V-1}(A) = \frac{V E_V(A)}{A(1 - E_V(A))}, \tag{39}$$

where the initial solution, for $V = -1$, we can get on the basis of the following equation:

$$E_{-1}(A) = [-Ei(-A)A\mathrm{e}^{A}]^{-1}, \tag{40}$$

in which function $Ei(A)$ is defined as follows:

$$Ei(x) = -\int_x^\infty (At+A)^{-1}\mathrm{e}^{At+A}\mathrm{d}(At+A). \tag{41}$$

It is also possible to approximate the function (40) by the the following polynomial [29]:

$$E_{-1}(A) \approx \frac{b_0 + b_1 A + b_2 A^2 + b_3 A^3 + b_4 A^4}{a_0 + a_1 A + a_2 A^2 + a_3 A^3 + a_4 A^4}, \tag{42}$$

where:

$$\begin{aligned} a_0 &= 0,2677737343, & b_0 &= 3,9584969228,\\ a_1 &= 8,6347608925, & b_1 &= 21,0996530827,\\ a_2 &= 18,0590169730, & b_2 &= 25,6329561486,\\ a_3 &= 8,5733287401, & b_3 &= 9,5733223454,\\ a_4 &= 1, & b_4 &= 1. \end{aligned}$$

Having at our disposal the values of fictitious traffic A_j^* and the equivalent group capacity V_j^*, we can calculate on the basis of (8) and (9) the parameters of the traffic overflowing from the primary group servicing PCT2 traffic streams, i.e. the variance σ_j^2 and the mean value R_j:

$$R_j = A_j^* E_{(V_j/t_j)+V_j^*}(A_j^*), \tag{43}$$

$$\sigma_j^2 = R_j\left[A_j^*/(V_j/t_j + V_j^* + 1 - A_j^* + R_j) + 1 - R_j\right]. \tag{44}$$

Let us notice that in Equation (43) and (44) the real link capacity V_j is divided by t_j because in the process of obtaining the capacity of fictitious link V_j^* we consider single-rate traffic (calls of each traffic class can demand only one BBU).

Having at disposal the parameters of traffic overflowing from primary groups, we can determine the occupancy distribution in the alternative group on the basis of the modified Kaufman-Roberts recursion, described in Section 3.3.

4. Modeling of systems with overflow multi-service traffic

In the previous section we dealt with the determination of the occupancy distribution in the alternative full-availability groups in systems in which primary groups serviced only one calls stream. This was purely theoretical case and its main purpose was to facilitate understanding of the introduced analytical dependencies. In real systems, primary groups carry multi-service traffic that is composed of several classes of calls.

The assumption that has been used so far allowed us to determine the variance of traffic that overflows from primary groups in a simple way through the application of Riordan formulas. With the case when the group carries multi-service traffic, direct application of the Riordan formulas is not possible. In this section we will present an approximate method for determining variances of different traffic streams that overflow from groups servicing multi-service traffic.

Let us consider the fragment of a multi-service network shown in Figure 6. The system is composed of v primary high-usage groups. Each of the group $s = 1, \dots, v$ is offered $m_{I,s}$ PCT1 traffic streams and $m_{J,s}$ PCT2 traffic streams ($m_s = m_{I,s} + m_{J,s}$). Calls of class c demand t_c BBUs to set up a connection[3]. The intensity of PCT1 traffic stream of class i offered to the group s is $A_{i,s}$. The intensity of PCT2 traffic offered by a single idle source of class j in the group s is $\alpha_{j,s}$, while the intensity of traffic $A_{j,s}(n)$ offered by all idle PCT2 sources of class j in the group s depends on the occupancy state n of the group in the following way:

$$A_{j,s}(n) = (N_{j,s} - n_{j,s}(n))\alpha_{j,s}, \qquad (45)$$

where $n_{j,s}(n)$ is the number of in-service sources of class j in the state of n BBUs being busy.

The traffic of particular classes, which is blocked in primary groups overflows to the alternative group. The blocking coefficient for calls of class i (PCT1) in the direct group s ($E_{i,s}$) can be determined on the basis of the Kaufman-Roberts formulas (25) and (26).

In the case of the full-availability group with PCT2 traffic stream, the Kaufman-Roberts recursion (25) can be rewritten in the form that includes characteristics of Engset traffic streams, namely:

$$n[P_n]_{V_s} = \sum_{j=1}^{m_{J,s}} A_{j,s}(n - t_j) t_j [P_{n-t_k}]_{V_s}. \qquad (46)$$

According to the considerations presented in [9], the parameter $n_{j,s}(n)$ in Equation (45) can be approximated by the so-called *reverse transition rate* and can be calculated on the basis of the local equations of equilibrium [19, 30]:

$$n_{j,s}(n) = \begin{cases} A_{j,s}(n-t_j)[P_{n-t_j}]_{V_s} \big/ [P_n]_{V_s} & \text{for } n \le V_s, \\ 0 & \text{for } n > V_s. \end{cases} \qquad (47)$$

The reverse transition rate determines the average number of class j calls serviced in the state n. Let us note that to determine the parameter $n_{j,s}(n)$ the knowledge of the occupancy distribution $[P_n]_{V_s}$, is necessary. In order to determine the distribution $[P_n]_{V_s}$ in turn, it is necessary to know the value $n_{j,s}(n)$. Equations (47) and (46) form then a set of confounding equations that can be solved with the application of iterative methods. In line with [9], in the first iteration we assume that the parameters $\forall_{j \in m_j} \forall_{0 \le n \le V} \; n^{(0)}_{j,s}(n) = 0$. The adopted assumption means that the Engset streams – in the first iteration – can be treated as an equivalent Erlang streams generating the offered traffic with the intensity:

[3] In the paper it is assumed that the letter "i" denotes a Poisson (Erlang) traffic class, the letter "j" – a Binomial (Engset) traffic class, and the letter "c" – an arbitrary traffic class, ($c = i|j$)

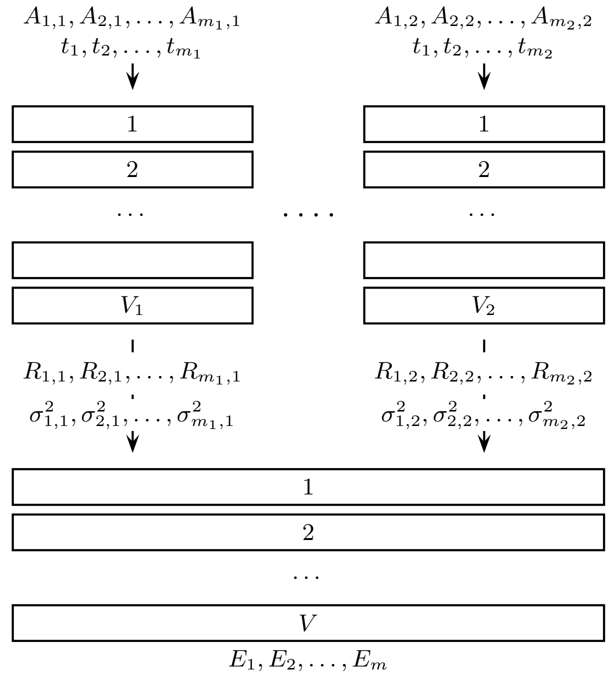

Figure 6. A fragment of telecommunications network with overflow multi-service traffic

$$A_{j,s}(n) = A_{j,s} = N_{j,s}\alpha_j, \qquad (48)$$

which is equal in value to the traffic offered by all free sources of class j Engset stream. The state probabilities, obtained on the basis of Eq. (46), constitute the input data for the next iteration l, where the parameters $n^{(l)}_{j,s}(n)$ and subsequently $A_j(n)$ are designated. The iterative process ends when the assumed accuracy ϵ is obtained:

$$\forall_{j \in \langle 1, m_J \rangle} \forall_{n \in \langle 0, V \rangle} \left(\left| \frac{n^{(l-1)}_{j,s}(n) - n^{(l)}_{j,s}(n)}{n^{(l)}_{j,s}(n)} \right| \le \epsilon \right). \qquad (49)$$

The obtained occupancy distribution $[P_n]_{V_s}$ in the group with Engset traffic streams allows us to calculate the blocking probability $E_{j,s}$ on the basis of Equation (26).

Knowing blocking probabilities for PCT1 and PCT2 streams we are in position to determine the mean value of the intensity of class c traffic that overflows from the group s:

$$R_{c,s} = A_{c,s}E_{c,s}. \tag{50}$$

To characterize overflow traffic fully it is necessary to determine the variance of each of calls streams. This parameter will be determined in an approximate way by carrying out a decomposition of each of the real groups into m_s fictitious component groups with the capacities $V_{c,s}$. Each fictitious group will be servicing exclusively calls of one class, which will make it possible to apply the Riordan formulas to determine the variance $\sigma^2_{c,s}$ of the traffic of class c that overflows from the group s. Let us determine then the capacities of the fictitious groups. For this purpose we first determine the carried traffic of class c in the group s:

$$Y_{c,s} = A_{c,s}(1 - E_{c,s}). \tag{51}$$

According to the definition, the value $Y_{c,s}$ defines the average number of calls of class c serviced in the group s. Therefore, the mean value of the intensity of class c traffic, expressed in BBUs, will be equal to $Y_{c,s}t_c$. The capacity of a fictitious component group $V_{c,s}$ will be defined as this part of the real group V_s which is not occupied by calls of the remaining classes (different from class c). Thus, we get [10–12]:

$$V_{c,s} = V_s - \sum_{l=1;l\neq c}^{m_{I,s}+m_{J,s}} Y_{l,s}t_l, \tag{52}$$

where V_s is the capacity of the primary group and the sum on the right side of Equation (52) determines the number of BBUs occupied by the calls of the remaining classes. The proposed decomposition allows us to use the method proposed in Section 3.4, to convert the system with PCT2 traffic streams to the equivalent PCT1 traffic streams.

Having all the parameters at our disposal for PCT1, i.e. $R_{i,s}$, $A_{i,s}$, $V_{i,s}$ and PCT2, i.e. $A^*_{j,s}$, $R_{j,s}$, $V^*_{j,s}$, $V_{j,s}$ we can – on the basis of the Riordan formula – determine the variance $\sigma^2_{i,j}$ for individual calls streams that overflow to the alternative group:

$$\sigma^2_{i,s} = R_{i,s}\left[\frac{A_{i,s}}{V_{i,s}/t_i + 1 - A_{i,s} + R_{i,s}} + 1 - R_{i,s}\right], \tag{53}$$

$$\sigma^2_{j,s} = R_{j,s}\left[\frac{A^*_{j,s}}{V_{j,s}/t_j + V^*_{j,s} + 1 - A^*_{j,s} + R_{j,s}} + 1 - R_{j,s}\right], \tag{54}$$

where the quotient $V_{c,s}/t_c$ normalizes the system to a single-service case. Such an operation is necessary since the Riordan formulas in their basic form are designed for determining overflow traffic parameters in single-service systems.

Since individual calls streams offered to the system are statistically independent, then the parameters of the total traffic of class c offered to the alternative group will be equal to:

$$R_c = \sum_{s=1}^{v} R_{c,s}, \quad \sigma^2_c = \sum_{s=1}^{v} \sigma^2_{c,s}. \tag{55}$$

At this point we have all the parameters that characterize m calls streams offered to the alternative group. Having at our disposal the dependencies (55), we can determine the occupancy distribution and the blocking probability in the system with overflow multi-service traffic shown in Figure 6. In order to do that, we can apply the formulas (28) and (29), where the overall coefficient Z is determined according to Equation (30).

Summing up our considerations, we can present the process of determining occupancy distribution in the alternative group of hierarchically organised networks with overflow traffic in the form of the Algorithm Overflow-MKRR.

Algorithm 3 Algorithm Overflow-MKRR

1. Determination of blocking probability of class $c = 1, \ldots, m$ calls stream in each of primary groups v;
2. Determination of the mean value $R_{c,s}$ of class c traffic overflowing from the primary group $s = 1, \ldots, v$;
3. Decomposition of the primary group s (with the capacity of V_s BBUs), servicing m_s traffic classes, on the m_s groups where each has the capacity of $V_{c,s}$ BBUs (Equation (52));
4. Conversion of PCT2 traffic stream to the equivalent PCT1 traffic stream (Section 3.4);
5. Determination of the variance $\sigma^2_{c,s}$ of class c traffic stream overflowing from the primary group $V_{c,s}$ to the alternative group V_{alt} (Equation (53) and (54));
6. Determination of the parameters of class c overflow traffic offered to the alternative group (Equation (55));
7. Determination of the overall coefficient Z (Equation (30));
8. Determination of the occupancy distribution in the alternative group (Equation (28));
9. Determination of blocking probability for all traffic classes in the alternative group (Equation (29)).

5. Numerical examples

The presented methods for determining the parameters of overflow traffic, the occupancy distribution and the blocking probability in systems with overflow multi-service

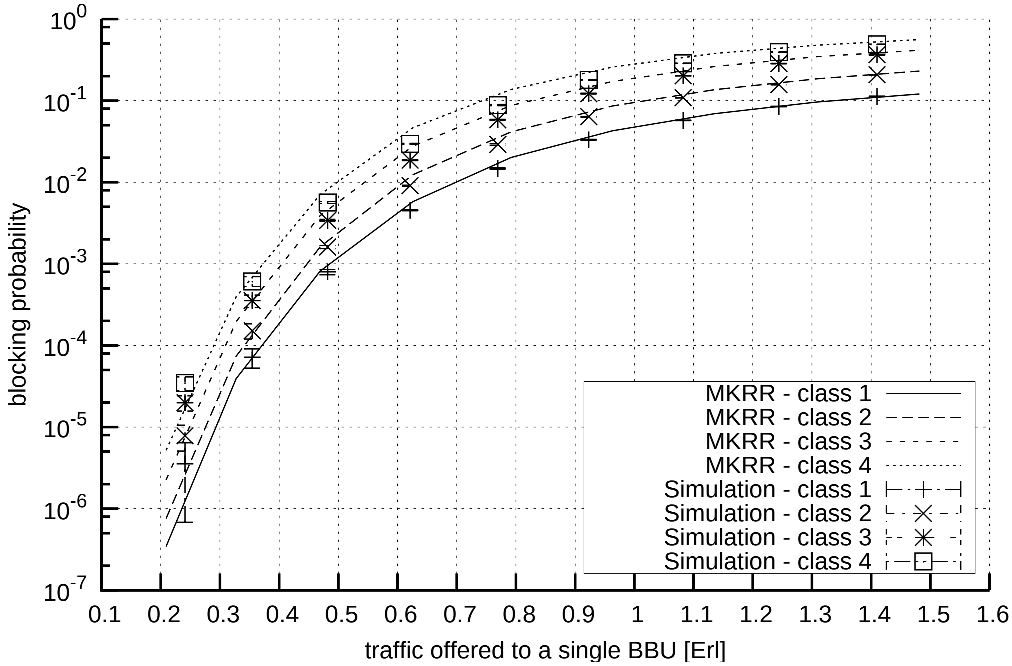

Figure 7. Blocking probability in the alternative group with overflow multi-service traffic with capacity equal to $V = 200$ BBUs; first and second primary groups: $V_1 = V_2 = 60$ BBUs, $t_1 = 2$ BBUs, $t_2 = 4$ BBUs, $t_3 = 8$ BBUs $A_{1,1}t_1 : A_{2,1}t_2 : A_{3,1}t_3 = 1 : 1 : 1$, $A_{1,2}t_1 : A_{2,2}t_2 : A_{3,2}t_3 = 1 : 1 : 1$; third and fourth primary groups: $V_3 = V_4 = 100$ BBUs, $t_1 = 2$ BBUs, $t_2 = 4$ BBUs, $t_3 = 8$ BBUs, $t_4 = 12$ BBUs, $A_{1,3}t_1 : A_{2,3}t_2 : A_{3,3}t_3 : A_{4,2}t_4 = 1 : 1 : 1 : 1$, $A_{1,4}t_1 : A_{2,4}t_2 : A_{3,4}t_3 : A_{4,4}t_4 = 1 : 1 : 1 : 1$; fifth primary group: $V_5 = 40$ BBUs, $t_2 = 4$ BBUs

traffic are the approximate methods. To determine the precision of the proposed solution, results of analytical calculations were compared with the simulation data. The research was carried out for two networks. The first network was composed of five primary groups servicing multi-service PCT1 (Erlang) traffic streams and one alternative group (with the capacity of 200 BBUs) servicing the traffic overflowing from the primary groups. The second network was composed of three primary groups servicing multi-service PCT2 (Engset) traffic streams and one alternative group (with the capacity of 100 BBUs) servicing the overflowed traffic.

The parameters of the offered traffic and the capacities of individual groups are given in the captions to Figures 7 and 8 presenting the obtained blocking probability results in the alternative group – both analytical and simulation results. The value of the blocking probability is expressed in the function of normalized traffic a offered to a single BBU of the alternative group:

$$a = \frac{\sum_{c=1}^{m} R_c t_c}{V_{\text{alt}}}. \tag{56}$$

It was assumed that there was equal the normalized traffic u offered per single BBU in each of v direct groups:

$$\forall_{1 \le s \le v} \; u = \sum_{c=1}^{m} \frac{A_{c,s} t_c}{V_s}. \tag{57}$$

The simulation results are shown in Figures 7 and 8 in the form of appropriately denoted points with 95-percent confidence interval, calculated according to the t-Student distribution for 5 series, with 1000000 calls of each class.

On the basis of the obtained blocking probability results in the considered systems we can state that the proposed calculational method for overflow traffic parameters combined with the modification of Kaufman-Roberts formula (28) provides high accuracy of calculations.

6. Conclusion

An analytical method for determining the occupancy distribution and blocking probability in groups of telecommunication networks servicing overflow multi-service traffic is presented in the article. The presented method is based on a modification of the Kaufman-Roberts formula,

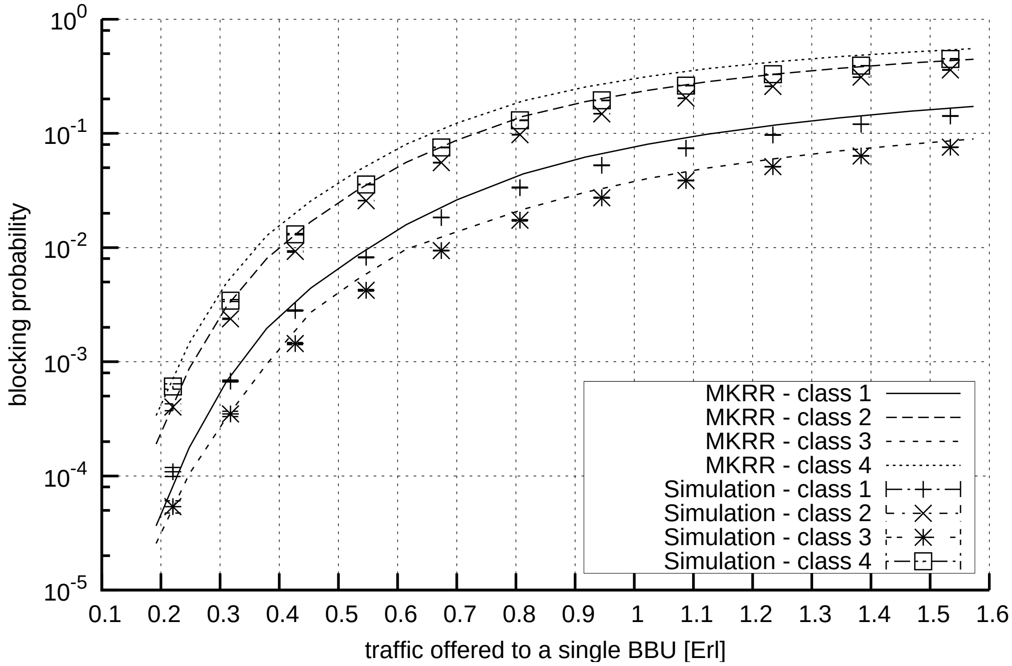

Figure 8. Blocking probability in the alternative group with overflow multi-service traffic with capacity equal to $V = 100$ BBUs; first primary group: $V_1 = 60$ BBUs, $t_2 = 2$ BBUs, $S_2 = 80$, $t_3 = 6$ BBUs, $S_3 = 60$, $A_{2,1}t_2 : A_{3,1}t_3 = 1 : 1$; second primary groups: $V_2 = 80$ BBUs, $t_1 = 1$ BBUs, $S_1 = 100$, $t_4 = 8$ BBUs, $S_4 = 60$, $A_{1,2}t_1 : A_{4,2}t_2 = 1 : 1$; third primary group: $V_3 = 100$ BBUs, $t_1 = 4$ BBUs, $S_1 = 100$, $t_3 = 6$ BBUs, $S_3 = 60$, $t_4 = 8$ BBUs, $S_4 = 60$

which involves an introduction of the peakedness coefficient Z that characterizes the unevenness of the overflow calls stream. Additionally, an analytical method for determining the occupancy distribution and blocking probability in groups of telecommunication networks servicing overflow multi-service traffic with a finite as well as infinite number of traffic sources is presented in the article. The presented method is based on conversion of traffic streams, generated by finite source population, to the traffic streams, generated by infinite source population. The accuracy of the proposed analytical method is verified by the presented simulation data.

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