On Type II Hybrid-ARQ with Decode and Forward Relay using Non-Binary Rate-Compatible Punctured LDPC Code on MIMO Frequency Selective Channels

Sho Kato and Yasunori Iwanami
Dept. of Computer Science and Engineering
Nagoya Institute of Technology
Nagoya, Japan
E-mail: 26417542@stn.nitech.ac.jp, iwanami@nitech.ac.jp

Abstract— **In this paper, Non-Binary Rate-Compatible Punctured Low Density Parity Check (NB RCP LDPC) code is designed over the extended Galois Field. The designed NB RCP LDPC code is applied to the type II Hybrid Automatic Repeat reQuest (HARQ) with Decode and Forward (DF) relay on Multiple Input Multiple Output (MIMO) Orthogonal Frequency Division Multiplexing (OFDM) channel and MIMO Single Carrier-Frequency Division Multiple Access (SC-FDMA) channel. The designed code enables us to decrease the coding rate with incremental redundancy for each retransmission in HARQ. The retransmission is done from the DF relay after the successful decoding in the relay. We have verified through computer simulations that the proposed type II HARQ scheme with DF relay greatly improves the throughput and average retransmission characteristics compared with the scheme without DF relay. Multiple relay cases are also considered.**

Keywords-NB RCP LDPC code; Hybrid-ARQ; Decode and Forward Relay; MIMO; OFDM; SC-FDMA; Symbol-LLR.

I. INTRODUCTION

An LDPC code that suits the flexible coding rate design and has the high error correcting capability through iterative decoding can be constructed on arbitrary extended Galois Field (GF). The Non-Binary (NB) LDPC code constructed on extended GF [1],[2] generally exhibits the better Bit Error Rate (BER) performance than the binary LDPC codes [3],[4]. There also exist RCP LDPC codes with variable coding rate obtained by properly puncturing the mother LDPC code [5]. The RCP LDPC codes enable us to use the same decoder as the mother code and suit the ARQ error correcting schemes [6],[7] with the incremental redundancy. By combining the NB LDPC codes with the RCP codes, the NB RCP LDPC codes were designed and the designed NB RCP LDPC codes were applied to the type II HARQ [8],[9]. When comparing the HARQ using NB RCP LDPC codes with the existing RCPT (Rate Compatible Punctured Turbo) HARQ using binary Turbo codes [10], the HARQ with NB RCP LDPC codes can cope with flexible coding rates, code word lengths and NB symbol LLR [11] additions without using inter-leavers for burst errors on the channel. On the other hand, the Decode and Forward (DF) relay schemes [12],[13] are useful for HARQ schemes. By using the DF relay, the source node can be replaced by the relay, once the relay correctly decodes the packet from the source. This replacement from the source to the relay effectively reduces the number of retransmissions and improves the throughput. In [9], NB LDPC coding with NB repletion codes is applied to multiple relay case for flat fading channel. The NB RCP LDPC coded type II HARQ with DF relay is applied to the MIMO-OFDM modulation in [2]. The incremental redundancy in HARQ with DF relay is also suited to the up-link transmission like in Long Term Evolution (LTE) or 4G. Due to the necessities of low PAPR and the high power efficiency in the amplification, MIMO SC-FDMA [14] is usually adopted to the up-links in cellular networks. Among SC-FDMA, interleaved SC-FDMA is especially useful because of its very low PAPR nature and excellent frequency diversity effect [14]. The application of NB RCP LDPC codes to MIMO interleaved SC-FDMA with multiple DF relays was reported in [1].

In this paper, we have examined in detail the performance of NB RCP LDPC coded type II HARQ with DF relays on MIMO OFDM channel and MIMO interleaved SC-FDMA channel with completely new simulations. Although some parts in this paper are identical to [1] and [2], the contents of [1] and [2] are verified and some new simulations and results are added. We have confirmed that the proposed HARQ scheme with DF relay greatly improves the throughput and the average number of retransmission characteristics compared with the case of no DF relay. Moreover, we have investigated the multiple relay cases.

The paper is organized as follows. In Section II, RCP LDPC code is introduced. In Section III, NB LDPC coded Type II HARQ scheme is described. In Section IV, we introduce the DF relaying scheme. In Section V, we briefly illustrate the MIMO OFDM and MIMO interleaved SC-FDMA modulation. In Section VI, computer simulation results are shown. The paper concludes with Section VII.

II. RCP LDPC CODE

The encoding and decoding procedure of RCP LDPC code is as follows. We call the code before puncture and the code after puncture as the mother code and the efficient code, respectively. In RCP LDPC code, the encoder and decoder of mother code can also be applied to the efficient code. When the parity check matrix of mother code is given by $\boldsymbol{H}_M(n_M \times n_N)$ and the generator matrix by $\boldsymbol{G}_M(n_N \times n_K)$ with $n_K = (n_N - n_M)$, the coding rate of mother code becomes $R_M = (1 - n_M / n_N) = n_K / n_N$. The coding rate after the puncture of n_P symbols from the mother code is given by $R_E = n_K / (n_N - n_P)$. We denote the message vector as $\boldsymbol{m} = (m_1, m_2, \ldots, m_{n_K})$, the code word of mother code as $\boldsymbol{C_M} = (c_{M1}, c_{M2}, \ldots, c_{Mn_N})$, the index of position to be punctured as $\boldsymbol{P} = (p_1, p_2, \ldots, p_{n_P})$ and the code word of punctured code as $\boldsymbol{C}_P = (c_{P1}, c_{P2}, \cdots, c_{Pn_P})$. The encoding procedure is first to generate the mother code by $\boldsymbol{C_M} = \boldsymbol{mG_M}$ which is systematic, and next, to puncture the position using $\boldsymbol{P}$ to obtain $\boldsymbol{C}_P$. The decoding procedure is to produce the symbol LLR (Log Likelihood Ratio) [11] from the receive signal and it is fed to the mother code decoder as the initial value for the sum-product algorithm. The symbol LLR for the position $\boldsymbol{P}$ is initially set to 0, because there is no available symbol LLR corresponding to the position $\boldsymbol{P}$.

III. NB RCP LDPC CODED TYPE II HARQ SCHEME

In Figure 1, we show the transmitter and receiver block diagram of NB RCP LDPC coded Type II HARQ. At the transmitter, the data bits are firstly encoded by the Cyclic Redundancy Check (CRC)-16 error detecting code and secondly encoded by the NB LDPC code on GF(4) or GF(16). The encoded LDPC code word is divided into the transmission packets and they are modulated by Quaternary Phase Shift Keying (QPSK) or 16 Quadrature Amplitude Modulation (16QAM) depending on GF(4) or GF(16), respectively. Matching GF(Q) to the modulation level Q is preferable in calculating the symbol LLR and reduces the complexity compared with the use of bit LLR calculation. After the interpolation filtering and the up-conversion to carrier frequency, the Radio Frequency (RF) signal is transmitted from the antenna.

At the receiver side, the received signal is demodulated and the symbol LLR is calculated. The symbol LLR is then fed to the NB LDPC decoder. Using Sum-Product Algorithm (SPA), the LDPC code word is decoded and the hard decision is made to obtain the data bits. The data bits

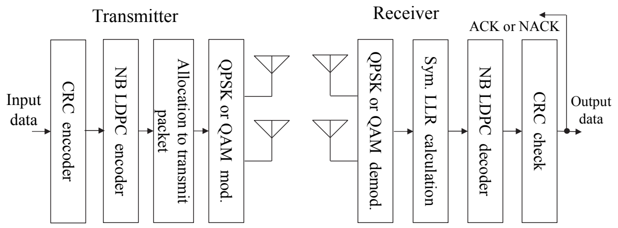

Figure 1. Transmitter and receiver structure of NB RCP LDPC coded type II HARQ scheme

TABLE I. SIMULATION CONDITIONS OF RCP LDPC CODE

Channel		AWGN	
Modulation		QPSK	16QAM
Size of Galois field		GF(4)	GF(16)
Mother code	Size of parity check matrix	(256,512)	(128,256)
	Average weight	(2.66,5.32)	(2.41,4.82)
	Coding rate	4/8	2/4
Efficient code	Information bit length	512	
	Coding rate	4/8,4/7,4/6,4/5, 4/4	2/4,2/3,2/2
Max SPA iteration		20	

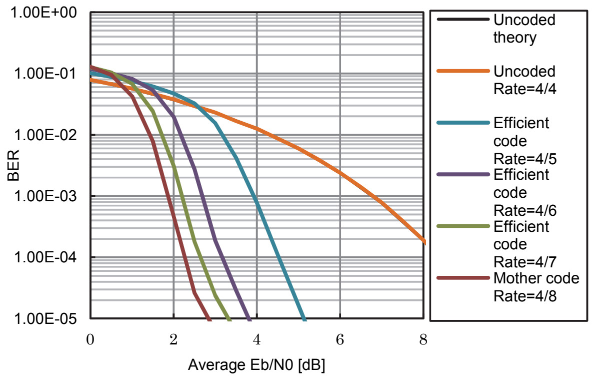

Figure 2. BER characteristics of NB RCP-LDPC code on AWGN channel (QPSK, Uncoded theory and uncoded rate 4/4 are overlapped.)

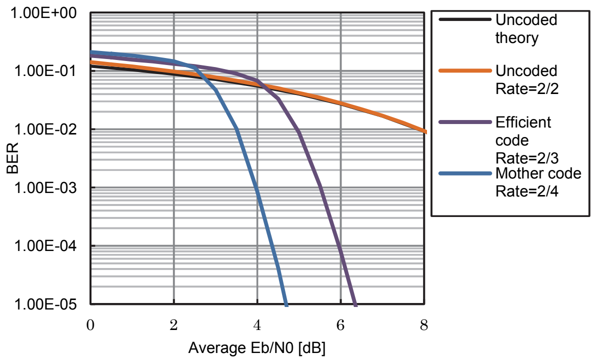

Figure 3. BER characteristics of NB RCP-LDPC code on AWGN channel (16QAM, Uncoded theory and uncoded rate 2/2 are overlapped.)

are then CRC-checked and Negative ACKnowledgement (NACK) or ACKnowledgement (ACK) is returned to the transmitter corresponding to the error or no error detection. The BER characteristics of RCP LDPC code on Additive White Gaussian Noise (AWGN) channel are examined when the rate 1/2 mother code on GF(4) or GF(16) is punctured to change the coding rate. The simulation condition is listed in Table I and the simulation results are shown in Figure 2 and Figure 3. C++ language is utilized for programming. From the simulation results, we know that the efficient codes on GF(4) or GF(16) with different coding rates are obtained from a mother code and the error correction capability corresponding to each coding rate is achieved.

In type II HARQ, like in Figure 4, at the first transmission, only uncoded information symbols are transmitted, and at the second transmission and after, the parity check symbols are retransmitted at the incremental redundancy policy. Accordingly, when the channel condition is good, the first uncoded transmission is successful and it achieves high throughput. On the other hand, when the channel condition is bad, by decreasing the coding rate at each retransmission, the error correction capability increases gradually. The generation of NB RCP LDPC code is done only once at the transmitter and there is no need of regeneration of code word when the coding rate

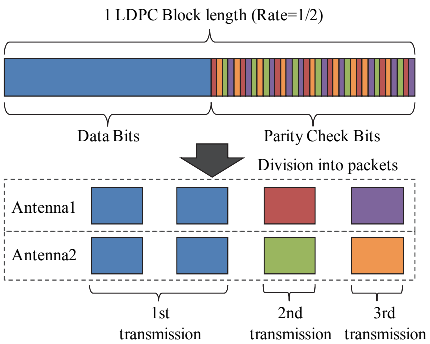

Figure 4. Division of NB LDPC code word into transmission packets when 2 spatial multiplex streams are employed

is decreased. Therefore, there is no increase of complexity of NB RCP LDPC code compared with the fixed rate NB LDPC code. Also at the receiver side, the complexity of NB RCP LDPC decoder does not increase compared with the fixed rate NB LDPC decoder, because the same and only one NB LDPC decoder can be used for various coding rates of NB RCP LDPC code.

IV. DECODE AND FORWARD RELAYING SCHEME

The Decode and Forward relay model is shown in Figure 5. We consider the relay arrangement where the relay locates at the middle point between the source (transmitter) and the destination (receiver).

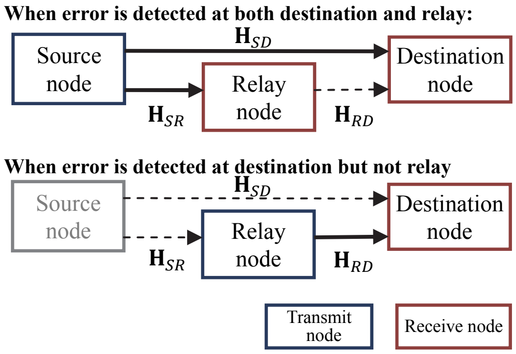

Figure 5. DF (Decode and Forward) relaying model

At the first transmission, the source broadcasts the uncoded information packet to the relay and destination simultaneously. The relay and destination independently detect the transmission errors using CRC-16 code. The relay and destination independently return (broadcast) ACK or NACK to the source. This ACK or NACK is shared among source, relay and destination. If the destination returns ACK, the transmission finishes at the first transmission and this condition is equivalent to no relay. Otherwise, retransmission is made. The source sends parity check packets with incremental redundancy. The relay and destination receive the parity check packet and combine it with already received packet. The LDPC decoding and CRC error detection is done both at relay and destination. ACK or NACK is returned and shared among source, relay and destination. At this point, if destination returns NACK but relay does ACK, then the relay sends the parity check packet hereafter instead of the source, i.e., the source is replaced by the relay which locates closer to destination. The transmission from relay to destination is more successful than source to destination due to the near distance between relay and destination. Also, as the source and relay do not simultaneously retransmit the parity check packet, the total transmission power is the same between with and without relay. This saves the total transmit energy in the case when the same transmit power as source is allocated to the relay.

Next, we consider the two relay cases where two relays are allocated in parallel or serial manner as shown in Figure 6(b) or (c), respectively. Figure 6 (a) is the arrangement of single relay already discussed. In Figure 6 (b), two relays are allocated in the middle point between source and destination in parallel. In Figure 6(c), relay 1 and relay 2 are allocated serially with equal distance interval between source and destination. When the power attenuation exponent is given by α and the distance between source and destination is normalized as 1, the relay at the middle point between source and destination in Figure 6 (a) and (b) receives 2^{α} times more power than the direct link between source and destination. Similarly, the relay 1 and relay 2 in Figure 6 (c) receives 3^{α} times more power than the direct link.

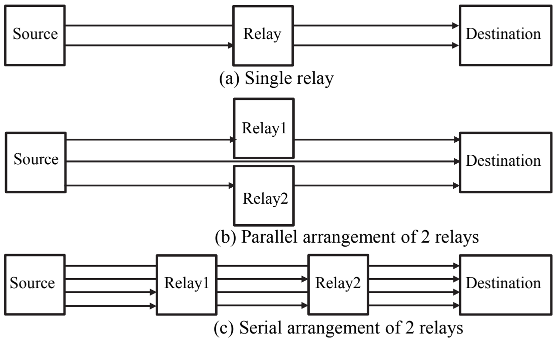

Figure 6. DF relay arrangement in case of multiple relays

V. MIMO OFDM AND MIMO SC-FDMA MODULATION SHEME

In Figure 7, we show the block diagram of NB RCP LDPC coded Type II HARQ scheme using MIMO-OFDM modulation. At the transmitter, the data bits are firstly encoded by the CRC-16 error detecting code and secondly encoded by the NB LDPC code on GF(4) or GF(16). The encoded NB alphabets are mapped to QPSK signal points for GF(4) or 16QAM for GF(16). These signal points are then modulated by OFDM with guard interval insertion for making a packet for each transmission. The OFDM signal is then transmitted to the frequency selective channel from each transmit antenna. At the receiver, for each antenna, the guard interval is first removed and then OFDM demodulation is made using Fast Fourier Transform (FFT). By demodulating each subcarrier QPSK modulated or

16QAM modulated, the symbol LLR is calculated.

We show how the symbol LLR is calculated for each demodulated subcarrier of OFDM with QPSK signaling. When the transmit signal, receive signal, signal points of QPSK and the subcarrier channel fading value are denoted as x, r, s_0,s_1,s_2,s_3 and h, respectively, the symbol LLR for the alphabets $a=0,1,2,3$ on GF(4) is defined as

$$\begin{aligned} LLR_a &= \log_e\left\{\frac{P(x=a|r\Delta r)}{P(x=0|r\Delta r)}\right\} = \log_e\left\{\frac{P(s_a,r\Delta r)/P(r\Delta r)}{P(s_0,r\Delta r)/P(r\Delta r)}\right\} \\ &= \log_e\left\{\frac{P(s_a,r\Delta r)}{P(s_0,r\Delta r)}\right\} = \log_e\left\{\frac{P(s_a)p(r\Delta r|s_a)}{P(s_0)P(r\Delta r|s_0)}\right\} \\ &= \log_e\left\{\frac{p(r|s_a)\Delta r}{P(r|s_0)\Delta r}\right\} = \log_e\frac{p(r|s_a)}{P(r|s_0)} \end{aligned} \tag{1}$$

where the priori probabilities are set to $P(s_0)=P(s_1)=P(s_2)=P(s_3)=1/4$, i.e., equal probabilities. In (1), $P(x=a|r\Delta r)$ denotes the probability that the transmit symbol x equals a when the receive signal point r falls in the small area $r\Delta r$ centered at r. $p(r|s_a)$ is the transition probability density function from $s_a \to r$ and is expressed as

$$p(r|s_a) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{|r-hs_a|^2}{2\sigma^2}\right) \tag{2}$$

where σ^2 is the variance of receive noise. Accordingly, the symbol LLR is calculated as

$$LLR_a = \log_e\left\{\frac{p(r|s_a)}{p(r|s_0)}\right\} = \frac{|r-hs_0|^2-|r-hs_a|^2}{2\sigma^2} \tag{3}$$

The symbol LLR values are then fed to the LDPC decoder and the iterative decoding using sum-product algorithm is done. The decoded information bits are error-detected by the CRC-16 code. If error is not detected, the data bits are fed to the data sink and the ACK is returned to the transmitter to finish the transmission. But if errors are detected, the NACK is returned and the retransmission is requested. As the type II HARQ scheme is employed, at the first transmission, only the data symbols without encoding are sent to the receiver. After the 2nd transmission, as shown in Figure 4, the parity symbols are sent several times with the incremental redundancy depending on the error detection status at the receiver. When the channel quality is good, the uncoded data packet for the first transmission succeeds with high probability leading to the high throughput performance. On the other hand, when the channel quality is bad, the parity packets are retransmitted several times till the LDPC code rate reaches the lowest one

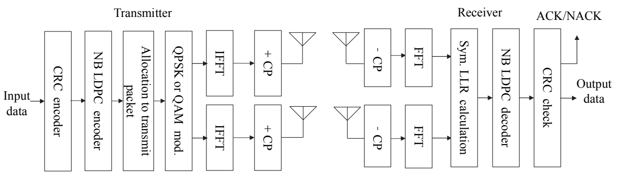

Figure 7. Transmitter and receiver configuration of NB RCP LDPC coded type II HARQ scheme using MIMO OFDM

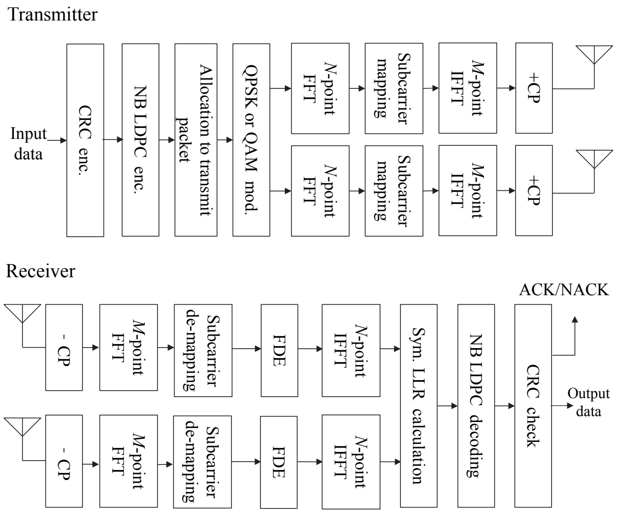

Figure 8. Transmitter and receiver configuration of NB RCP LDPC coded type II HARQ scheme using MIMO interleaved SC-FDMA

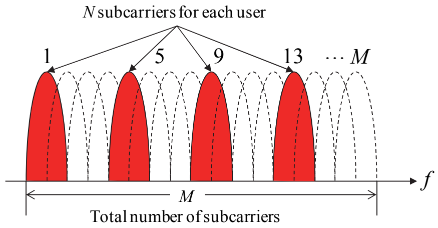

Figure 9. Subcarrier mapping in interleaved SC-FDMA

half resulting in enough error correction capability.

In Figure 8, we show the transmitter and receiver block diagram of NB RCP LDPC coded Type II HARQ using MIMO SC-FDMA. At the transmitter, the data bits are firstly encoded by the Cyclic Redundancy Check (CRC)-16 error detecting code and secondly encoded by the NB LDPC code on GF(4) or GF(16). The encoded LDPC code word is divided into the transmission packets and they are modulated by QPSK or 16QAM depending on GF(4) or GF(16), respectively. The modulated QAM symbols are then N-point FFT transformed at each antenna stream and the subcarrier mapping is done to make the interleaved SC-FDMA spectrum as shown in Figure 9. The interleaved spectrum is then M-point Inverse FFT (IFFT) transformed, where $M=U\times N$ and U is the number of users ($U=4$ in Figure 9). Cyclic Prefix (CP) is added to the time domain complex samples of an IFFT block. After the interpolation filtering and the up-conversion to carrier frequency, the RF signal is transmitted from each antenna. At the base station, after the down-conversion to baseband and the sampling, CP is removed and the M-point FFT is done to obtain the frequency domain signal. The frequency domain signal is then subcarrier-de-mapped to aggregate the interleaved spectrum of each user back to the N sample spectrum again. The Frequency Domain Equalization (FDE) is made to compensate the channel frequency response and separate the multiple spatial streams of each user. After the FDE, N-point IFFT is made to obtain the time domain signal of each stream. The subsequent symbol LLR calculation, NB LDPC decoding, CRC-16 error detection and the HARQ protocol follow the same manner as the MIMO OFDM above.

VI. COMPUTER SIMULATION RESULTS

The throughput performance and the average number of retransmission characteristic for 2×2 MIMO-OFDM NB RCP LDPC coded Type II Hybrid-ARQ with GF(4) and QPSK or with GF(16) and 16QAM are investigated. We compared the performance between with and without relay. The simulation condition is listed in Table II. The simulation results for throughput characteristic are shown in Figure 10 – Figure 13. The simulation results for average number of retransmission are shown in Figure 14 - Figure 17. We have also shown the simulation results for the parallel two relay case of Figure 6 (b) and the serial two relay case of Figure 6 (c) in Figures 18 and 19, respectively.

For QPSK, an LDPC code word on GF(4) is divided into 16 OFDM symbols. As the coding rate of mother LDPC code is 1/2, the former 8 OFDM symbols are the information data symbols and the latter 8 OFDM symbols are the parity check symbols. For the 1st transmission, 8 OFDM symbols made from information data are transmitted from 2 transmit antennas simultaneously using 4 OFDM symbol durations. For the 2nd transmission and thereafter, i.e., retransmission, 4 OFDM symbols made from the parity check symbols are transmitted from 2 transmit antennas simultaneously. In each retransmission, 1 parity check OFDM symbol is transmitted from each antenna using 1 OFDM duration. The coding rate is decreased gradually from 4/5, 4/6, 4/7 to 4/8 for each retransmission. For 16QAM, the coding rate is decreased from 2/2, 2/3 to 2/4. After all the parity check OFDM symbols are transmitted and the coding rate reaches 4/8=1/2, if the errors are still detected at the destination, the whole transmission of the same RCP LDPC code word in the same manner is repeated up to 15 times, which is enough large number to measure the throughput and the average number of retransmission. We call this procedure of decreasing the coding rate from 1 to 1/2 as the one set as shown in Table II. The symbol LLR addition is used at the destination for the repeated reception of the same RCP LDPC code word.

For the comparative scheme, we considered the type I HARQ with the fixed coding rate LDPC code and set the maximum number of retransmissions also to be 15 times.

We compare the throughput performance of type II HARQ with type I HARQ in Figures 10, 11, 12, and 13. As the coding rate of type I HARQ increases, the throughput also increases in the high average E_b/N_0 region. On the other hand, the throughput of type II HARQ approaches to 4 (bps/Hz) and 8 (bps/Hz) in case of QPSK and 16QAM, respectively, in the high E_b/N_0 region. This is because type II HARQ can change the coding rate adaptively and it can use the coding rate of 1 for high Signal to Noise Ratio

TABLE II. SIMULATION CONDITIONS OF NB GF(4) AND GF(16) RCP LDPC CODED TYPE II HYBRID ARQ SCHEME WITH 2×2 MIMO-OFDM

NB LDPC mother code	Size of Galois field	GF(4)	GF(16)
	Size of parity check matrix	(512,1024)	(256,512)
	Average weight	(2.66,5.32)	(2.41,4.82)
	Coding rate	4/8	2/4
Punctured code (efficient code)	Information bit length	1024	
	Coding rate	4/4,4/5,4/6, 4/7,4/8	2/2,2/3,2/4
Max SPA iteration		20	
Transmit and receive antennas		2 × 2	
Modulation		QPSK	16QAM
Number of OFDM subcarriers		N=64	
CP length (T_s :QAM symbol length)		$T_s/4$	
Channel model between each transmit and receive antenna		Quasi-static Rayleigh fading with 16 delay paths having equal average power	
Interval of delay paths		$T_s/64$	
Channel State Information		Known at receiver	
Error detecting code		CRC-16	
Power attenuation exponent		α=3	
Number of retransmission in Type I		15 times	
Number of retransmission in Type II		15 sets	15 sets

(SNR) region. The slight decrease of throughput in type II HARQ from 4 (bps/Hz) and 8 (bps/Hz) is due to the use of CRC-16 error detection code. In type II HARQ, however, the parity check packet is sequentially retransmitted in responding to the NACK, so the number of retransmission becomes large compared with the type I HARQ. Also in type II HARQ, the iterative decoding of LDPC code is done for each retransmission of parity check packet, thus the decoding time tends to increase.

Next, we compare the cases with and without relay. When the average receive E_b/N_0 is high, the throughputs with and without relay are almost equal, but when the average receive E_b/N_0 is low, the throughput with relay is higher than without relay. This is because for the high average receive E_b/N_0 region, the destination can receive the packet correctly without retransmission. Accordingly, the relay is not used for this high E_b/N_0 region, so there is no difference between with and without relay. On the other hand, for the region where the average receive E_b/N_0 is low, the transmission from source to destination often fails, but the transmission from relay to destination succeeds with high probability, thus the retransmission is switched from the source to the relay for this low E_b/N_0 region. For the type I HARQ schemes with a relay in Figure 11, Figure 13, Figure 15, and Figure 17, we know that the throughput with a relay is largely improved compared with the one without relay for the region where the average number of retransmission is 1. For this region the throughput of type I HARQ is almost one half of the throughput for high E_b/N_0 region. This means that for this region the transmission is switched from the source to relay and the retransmission from the relay to destination is almost successful. This observation proves that the use of

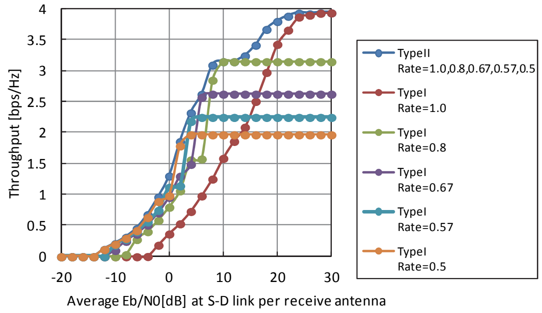

Figure 10. Throughput characteristics of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2, QPSK)

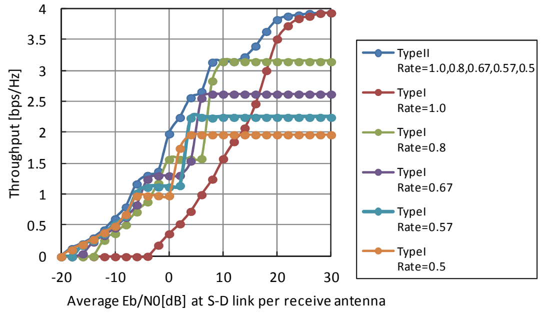

Figure 11. Throughput characteristics of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with a relay, 2×2, QPSK)

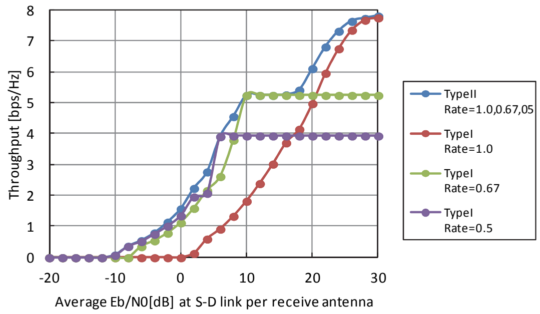

Figure 12. Throughput characteristics of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2, 16QAM)

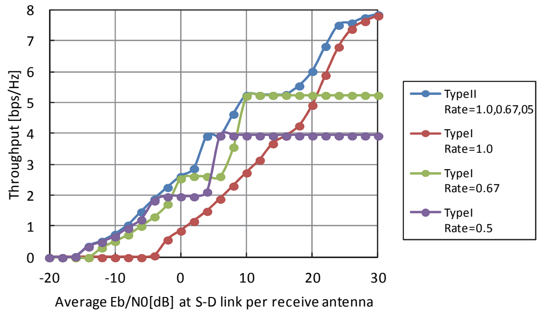

Figure 13. Throughput characteristics of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with a relay, 2×2, 16QAM)

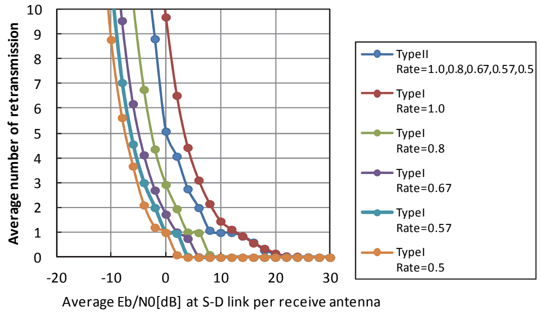

Figure 14. Average number of retransmissions of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2, QPSK)

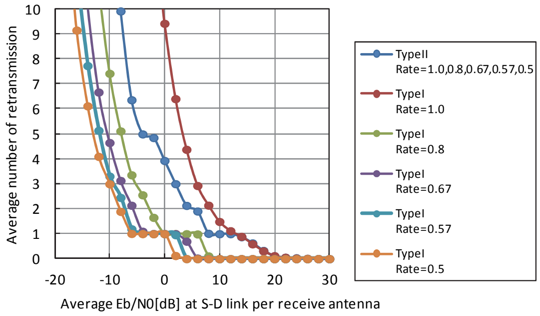

Figure 15. Average number of retransmissions of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with a relay, 2×2, QPSK)

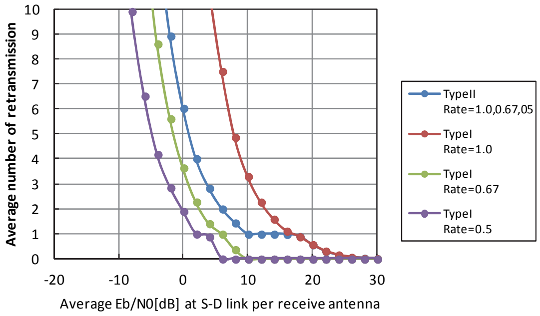

Figure 16. Average number of retransmissions of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2, 16QAM)

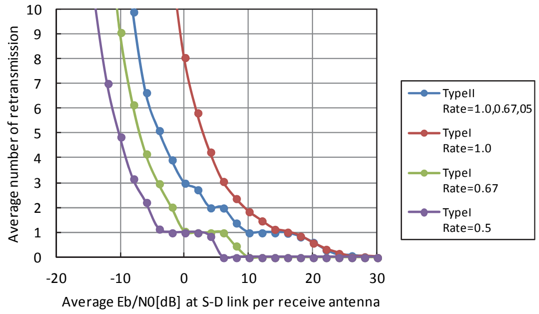

Figure 17. Average number of retransmissions of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with a relay, 2×2, 16QAM)

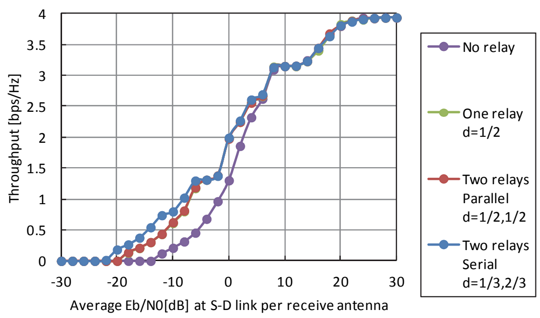

Figure 18. Comparison of throughput characteristics of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy for three relay arrangements (2×2 , QPSK, One relay d=1/2 and two relays parallel d=1/2, 1/2 are overlapped.)

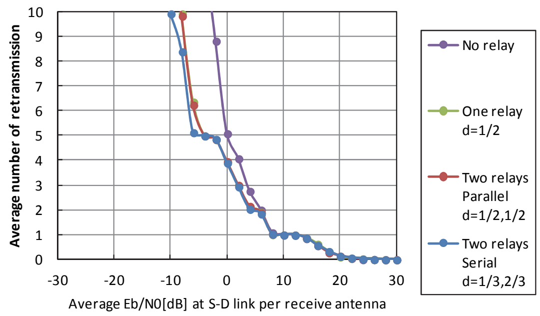

Figure 19. Comparison of average number of retransmissions of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy for three relay arrangements (2×2 , QPSK, One relay d=1/2 and two relays parallel d=1/2, 1/2 are overlapped.)

relay is quite effective in HARQ.

As for the proposed type II HARQ, the throughput is larger than all the type I HARQ schemes and is optimum for all average receive E_b/N_0 values. However, the average number of retransmission becomes larger than the type I ARQ schemes because of the incremental redundancy retransmissions.

As for the relay arrangements in Figure 6, from Figure 18 and Figure 19, we see that the throughput and the average number of retransmission characteristics for the serial arrangement in Figure 6 (c) show the best. The parallel arrangement in Figure 6 (b) exhibits almost the same performance as the one relay case in Figure 6 (a). This observation comes from the fact that the receive power at relay or destination in the serial arrangement becomes larger than the parallel arrangement.

Next, from Figure 20 - Figure 29, we show the throughput and the average number of retransmission characteristics of NB RCP LDPC coded type II HARQ on 2×2 MIMO interleaved SC-FDMA. The simulation conditions are listed in Table III. The simulation results for throughput characteristic are shown in Figure 20, Figure 21, Figure 22, and Figure 23. The simulation results for average number of retransmission are shown Figure 24, Figure 25, Figure 26, and Figure 27. We have shown the simulation results for the parallel two relay case and the serial two relay case in Figures 28 and 29.

When QPSK modulation is employed, one GF(4) LDPC mother code word is divided into 8 packets. As the coding rate of mother LDPC code is 1/2, the former 4 packets contain only information symbols and the latter 4 packets consist of parity check symbols. For the 1st transmission, 4 information packets are transmitted from two antennas. For the 2nd retransmission and after, 1 parity check packets are retransmitted at each retransmission resulting in lowering the coding rate at receiver from 4/5 to 4/6, 4/7, 4/8. After all the parity check packets are retransmitted and the coding rate at destination reaches 1/2, if error is still detected at destination, the same RCP LDPC code transmission is repeated 15 times and each time the symbol LLR's are summed up at destination by symbol LLR addition. We call this procedure of decreasing the coding rate from 1 to 1/2 as the one set. Thus, the total 15 sets of RCP LDPC code word transmission are done before the final discard of RCP LDPC code in case of failure of error correction at destination. As a comparative scheme, we also considered the LDPC coded type I HARQ scheme where the coding rate is fixed for each retransmission. The maximum number of retransmissions is limited to 15 and the symbol LLR addition is employed at the destination.

When 16QAM modulation is employed, one GF(16) LDPC mother code word is divided into 4 packets. The former 2 packets contain only information symbols and the latter 2 packets consist of parity check symbols. The coding rate decreases from 2/3 to 2/4 at each retransmission. After all the packets are retransmitted and the coding rate at destination reaches 1/2, if error is still detected at destination, the same RCP LDPC transmission is repeated 15 times in total, which is the same as the case of QPSK

TABLE III. SIMULATION CONDITIONS OF NB RCP LDPC CODED TYPE II HARQ WITH DF RELAY ON 2×2 MIMO INTERLEAVED SC-FDMA

NB LDPC mother code	Size of Galois field	GF(4)	GF(16)
	Size of parity check matrix	(512,1024)	(256,512)
	Average weight	(2.66,5.32)	(2.41,4.82)
	Coding rate	4/8	2/4
Punctured code (efficient code)	Information bit length	1024	
	Coding rate	4/4,4/5,4/6, 4/7,4/8	2/2,2/3,2/4
Max SPA iteration		20	
Number of users U		4	
Transmit and receive antennas		2×2	
Modulation		QPSK	16QAM
Number of subcarriers / user		$N=64$	
Number of total subcarriers		$M=256$	
CP length (T_s :QAM symbol length)		$16\times(T_s/4)=4T_s$	
Channel model between each transmit and receive antenna		Quasi-static Rayleigh fading with 16 delay paths having equal average power	
Interval of delay paths		$T_s/4$	
Channel State Information		Known at receiver	
Error detecting code		CRC-16	
Power attenuation exponent		$\alpha=3$	
Number of retransmission in Type I		15 times	
Number of retransmission in Type II		15 sets	15 sets

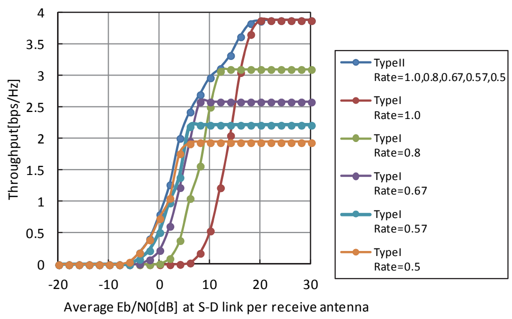

Figure 20. Throughput characteristics of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2 , QPSK)

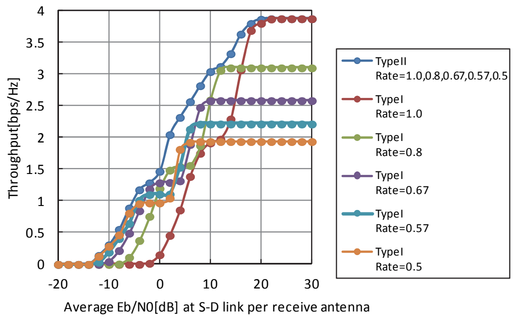

Figure 21. Throughput characteristics of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with a relay, 2×2 , QPSK)

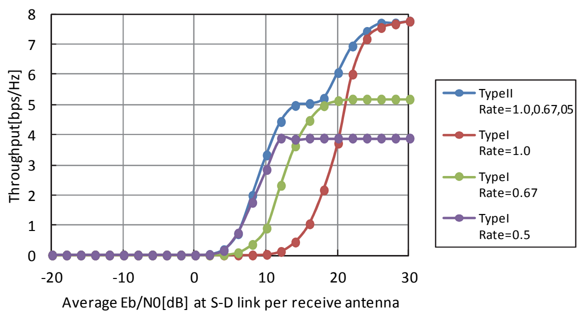

Figure 22. Throughput characteristics of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2 , 16QAM)

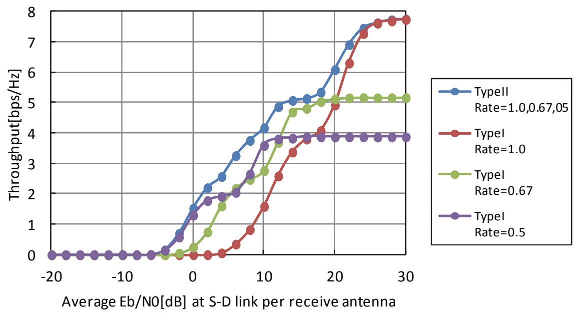

Figure 23. Throughput characteristics of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with a relay, 2×2 , 16QAM)

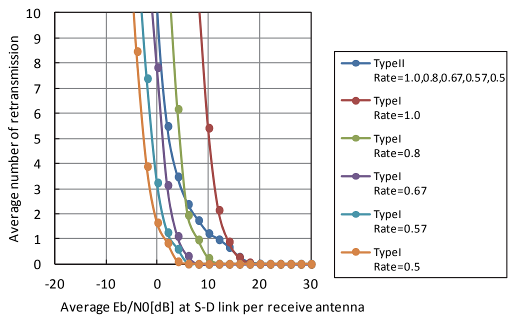

Figure 24. Average number of retransmissions of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2 , QPSK)

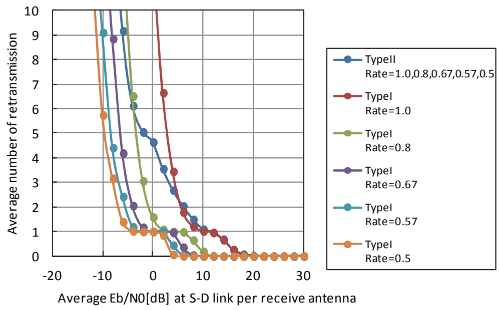

Figure 25. Average number of retransmissions of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with a relay, 2×2 , QPSK)

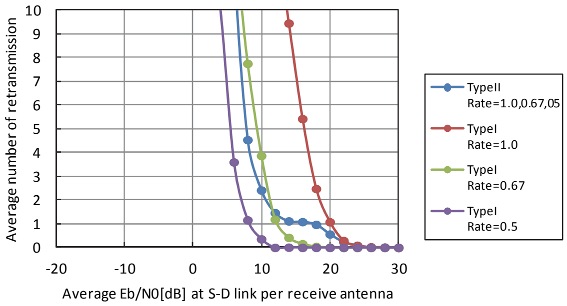

Figure 26. Average number of retransmissions of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2 , 16QAM)

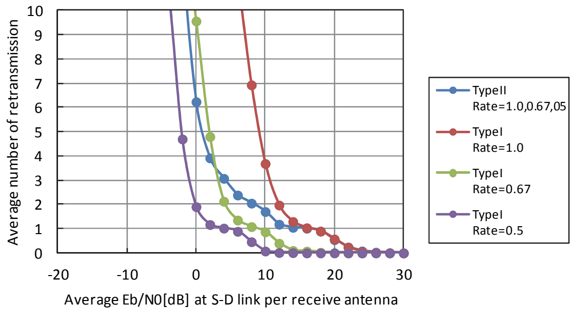

Figure 27. Average number of retransmissions of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with a relay, 2×2 , 16QAM)

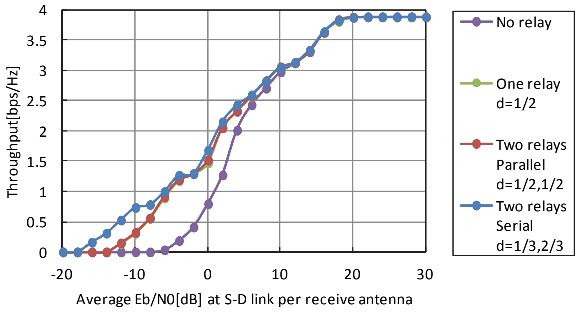

Figure 28. Comparison of throughput characteristics of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy for three relay arrangements (2×2 , QPSK, One relay d=1/2 and two relays parallel d=1/2, 1/2 are overlapped.)

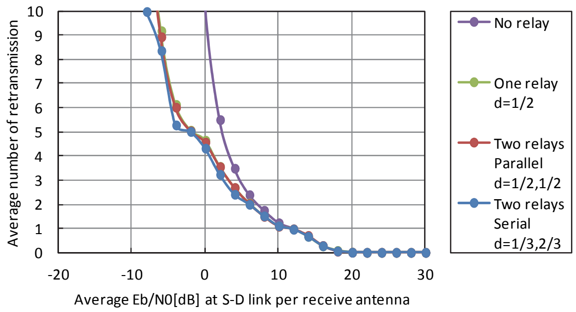

Figure 29. Comparison of average number of retransmissions of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy for three relay arrangements (2×2 , QPSK, QPSK, One relay d=1/2 and two relays parallel d=1/2, 1/2 are overlapped.)

modulation as mentioned in the above. As for the number of users U , we employed $U=4$, but any number of users U is available depending on how much subcarriers assigned to each user, i.e., $N=M/U$. Less number of subcarriers will lead to less frequency diversity effect.

For the two parallel relays or two serial relays, we also show the throughput and the average number of retransmission characteristics in Figure 28, and Figure 29, respectively.

Regarding the simulation results, we first compare the type I HARQ with the type II HARQ in case of no relay. The throughput characteristic of type I HARQ saturates in the high average E_b/N_0 region, because the coding rate is fixed. As the coding rate of type I HARQ increases, the throughput also increases in the high average E_b/N_0 region. On the other hand, the throughput of type II HARQ approaches to 4 (bps/Hz) and 8 (bps/Hz) in case of QPSK and 16QAM, respectively, in the high E_b/N_0 region. This is because type II HARQ can change the coding rate adaptively and it can use the coding rate of 1 for high SNR region. The slight decrease of throughput in type II HARQ is due to the use of CRC-16 error detection code. We also observe that for entire E_b/N_0 region, the throughput of type II HARQ is optimized and is superior to type I HARQ. However, the average number of retransmission of type II HARQ is worse than type I HARQ. This is because parity check packets are sent sequentially with several time slots in type II HARQ, while the parity check packet is sent at a

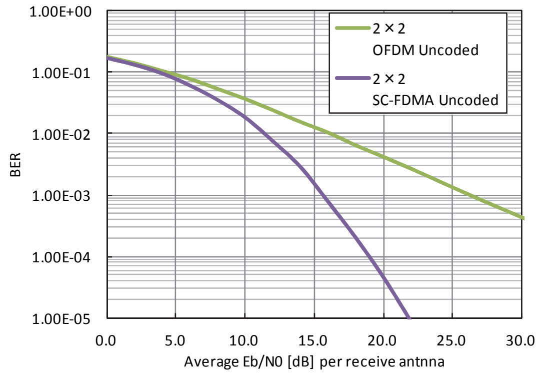

Figure 30. Comparison of BER characteristics between uncoded MIMO OFDM and MIMO interleaved SC-FDMA on 2×2 quasi-static multipath channel

time in type I HARQ.

Next, we compare the case with a relay and without relay. When the average E_b/N_0 is high, the throughput with a relay is the same as without relay. This is because, both in the case with a relay and without relay, the average number of retransmission is almost 0 for the high average E_b/N_0 region, and there makes no difference between the two. On the other hand, when the average E_b/N_0 is low, we see that the throughput and the average number of retransmission characteristics with a relay are much better than the ones without relay. This is because for low average E_b/N_0 region, although the destination frequently fails to decode the code word correctly, the relay succeeds in decoding with high probability. Accordingly, as the retransmission is executed from the relay to the destination instead of the source to the destination, the probability of successful decoding at destination is increased. We also observe that when the number of average retransmission is greater than 0, i.e., the retransmission is done and the total number of transmission is more than 2, the throughput with a relay is largely improved compared with the one without relay. This improvement is observed below $E_b/N_0\approx 20$ (dB) for GF(4) and QPSK in Figures 20, 21, 24, and 25, and below $E_b/N_0\approx 25$ (dB) for GF(16) and 16QAM in Figures 22, 23, 26, and 27.

On the relay arrangements in Figure 6, from Figure 28, and Figure 29, we observe that the throughput and the average number of retransmission characteristics for the serial arrangement in Figure 6 (c) show the best performance. The parallel arrangement in Figure 6 (b) exhibits almost the same performance as the one relay case in Figure 6 (a). This observation can be understood from the fact that the receive power at relay or destination in the serial arrangement becomes larger than the parallel arrangement as previously stated in MIMO OFDM.

Finally, we make the comparison between MIMO-OFDM and MIMO SC-FDMA with NB RCP LDPC coded type II HARQ. Almost the same observations we see between the two modulations. On the performance difference between the two modulations, we observe in Figure 10, Figure 14, Figure 20, and Figure 24 that the throughput performance and the average number of retransmission of MIMO interleaved SC-FDMA are better than the MIMO OFDM above $E_b / N_0 = 20$ (dB). This seems to be caused from the difference of BER performance between the two modulations. We show the BER characteristics of uncoded MIMO OFDM and uncoded MIMO interleaved SC-FDMA in Figure 30 simulated under the same simulation conditions of Table II and Table III. From Figure 30, we know the BER of MIMO interleaved SC-FDMA is better than the MIMO OFDM above $E_b / N_0 = 20$ (dB). Accordingly, the throughput and the average number of retransmission of MIMO interleaved SC-FDMA are superior to the MIMO OFDM.

In all the simulations in the above figures, a quasi-static Rayleigh fading channel with 16 delay paths having equal average power is considered and it is basically a static channel and time-invariant. If the mobile time-variant channel is considered and employed for the proposed schemes, further degradation of throughput and average number of retransmission is expected due to the channel estimation error caused by rapid time-varying channel. For future studies, the throughput and the average number of retransmission should be measured in real mobile environment using real test beds.

VII. CONCLUSIONS

In this paper, we have investigated the throughput and the average number of retransmission characteristics of the proposed NB RCP LDPC coded type II HARQ with DF relays using MIMO-OFDM and MIMO interleaved SC-FDMA. We have verified the effectiveness of the proposed scheme through computer simulation. In the proposed scheme, for the first transmission, only uncoded information packet is transmitted to both for relay and destination. If error is detected at destination, parity check packets are retransmitted for the 2nd and the subsequent retransmissions. The error correction decoding is done both at relay and destination. When the destination fails in decoding, but the relay succeeds, the relay replaces the source hereafter. The relay retransmits the remaining parity check packets with incremental redundancy instead of source. The destination receives the parity check packets till the coding rate reaches 1/2. We made clear that by using DF relays the throughput and the average number of retransmission characteristics are improved especially for low receive SNR region.

ACKNOWLEDGEMENT

This study is partially supported by the Grants-in-Aid for Scientific Research 15K06059 of Japan and the Sharp Corporation.

REFERENCES

[1] T. Hamada and Y. Iwanami, “On Type II Hybrid-ARQ with Decode and Forward Relay using Non-Binary Rate-Compatible Punctured LDPC Code on MIMO SC-FDMA up-link,” ICWMC 2014, pp. 112-117, June 2014.

[2] H. Tanaka and Y. Iwanami, “On Throughput Characteristics of Type II Hybrid-ARQ with Decode and Forward Relay using Non-Binary Rate-Compatible Punctured LDPC Codes,” ICWMC 2012, pp. 272-277, June 2012.

[3] D. Declercq and M. Fossorier, “Decoding algorithm for nonbinary LDPC codes over GF(q),” IEEE Transactions on Communications, vol. 55, pp. 633-643, April 2007.

[4] D. Kimura, F. Guilloud, and R. Pyndiah, “Application of non-binary LDPC codes for small packet transmission in vehicle communications,” The 5th International Conference on ITS Telecommunications, pp. 109-112, Brest France, June 2005.

[5] J. Ha, J. Kim, D. Klinc, and S. W. McLaughlin, “Rate-compatible punctured low-density parity-check codes with short block lengths,” IEEE Transactions on Information Theory, vol. 52, no. 2, pp. 728-738, Feb. 2006.

[6] M. Shimotsu, Y. Iwanami, and E. Okamoto, “An LDPC coded adaptive hybrid ARQ scheme with packet combining on MIMO eigen-mode channels,” IEICE Technical Report, RCS2005-37, pp.59-64, June 2005.

[7] Y. Tsuruta, Y. Iwanami, and E. Okamoto, “A Study on LDPC Coded Hybrid-ARQ Using Spatially Multiplexed MIMO-OFDM,” S36-1, 6 pages, WPMC2009, CD-ROM 5 pages, Sept. 2009.

[8] T. Kozawa, Y. Iwanami, E. Okamoto, R. Yamada, and N. Okamoto, “An evaluation on throughputs for Hybrid-ARQ using Non-Binary Rate-Compatible LDPC codes,” The 32nd Symposium on Information Theory and its Applications (SITA2009), F21-3, pp.771-775, Dec. 2009.

[9] T. Kozawa, Y. Iwanami, and E. Okamoto, R. Yamada, and N. Okamoto, “An evaluation on throughput performance for Type II Hybrid-ARQ using non-binary Rate-Compatible-Punctured LDPC codes,” IEICE Transactions on fundamentals, vol. E93, no.11, pp. 2089-2091, November 2010.

[10] D. Gang, R. Kimura, and F. Adachi, “Performance evaluation of RCPT Hybrid ARQ schemes for DS-CDMA mobile radio over frequency selective Rayleigh fading channel,” IEICE Technical Report, RCS2001-280, pp.241-248, March 2002.

[11] D. Declercq, V. Savin, and S. Pfletschinger, “Multi-Relay Cooperative NB-LDPC Coding with Non-Binary Repetition Codes,” ICN 2012, pp. 205-214, March 2012.

[12] J. Laneman, D. Tse, and G. Wornell, “Cooperative diversity in wireless networks: efficient protocols and outage behaviour,” IEEE Transactions on Information Theory, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[13] A. B. A. Aziz and Y. Iwanami, “A simple symbol estimation for soft information relaying in cooperative relay channels,” Int. Journal of Commun. Networks and Systems (IJCNS), Scientific Research Publishing, Vol. 4, No. 9, pp.568-577, Sept. 2011.

[14] H. G. Myung, J. Lim, and D. J. Goodman, “Single Carrier FDMA for Uplink Wireless Transmission,” IEEE Vehicular Technology Magazine, pp. 30-38, Sept. 2006.