Indoor Location Estimation Using Smart Antenna System with Virtual Fingerprint Construction Scheme

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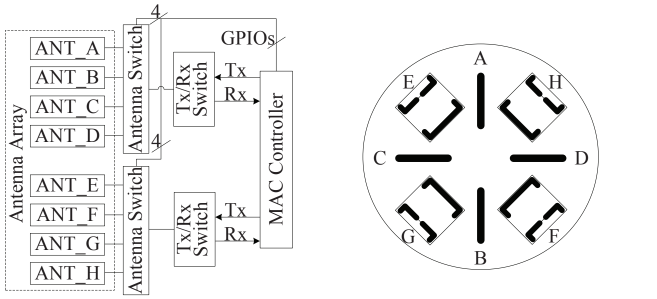
***Abstract*—Regarding indoor location estimation, many smart positioning techniques have been proposed and they could be classified into several categories. Fingerprinting is one of the categories and its features are to build the indoor radio map during offline phase and to utilize the radio map to estimate the location during online phase. Creating indoor radio map is the most critical step, where the accuracy of location estimation depends on the distribution and number of reference points (RPs). Usually, the time required to collect received signal strengths (RSSs) is proportional to the number of 'real' RPs. To reduce the complexity of offline process, this paper proposes an efficient scheme, which is based on the well-known signal propagation model, to construct 'virtual' RPs in indoor radio map. The RSSs on virtual RPs are calculated and used to substitute for the training RSSs on 'real' RPs collected during offline phase. The proposed scheme can not only shorten the collecting time but also achieve high accuracy for indoor positioning. The indoor location estimation using smart antenna system (SAS) also requires longer time to collect all RSS information because of multiple antennas. By combining this scheme with SAS, we can easily obtain enough and valid RSSs information to build the indoor radio map in a more efficient way. Experimental results showed that applying virtual fingerprint construction scheme on SAS can decrease 33% 'real' RPs in indoor radio map without sacrificing the positioning accuracy.**

Keywords - Fingerprint; Indoor Positioning; Location Estimation; Receive Signal Strength; Smart Antenna System; WLAN

I. INTRODUCTION

Nowadays, indoor location estimation is very important for many contemporary location-based services, such as health care monitoring, indoor navigation, personal tracking, inventory control, and so on. Without Global Positioning System (GPS) in indoor environment, there are many alternative techniques have been proposed [1][2] and they could be classify into several categories including Time of Arrival (ToA), Angle of Arrival (AoA) and the RSS-based location [3][4][5]. There are two reason why ToA and AoA are unsuitable for indoor environments: 1) they require the line-of-sight (LOS) between a pair of transmitter and receiver and 2) the special hardware design to support ToA and AoA algorithms is expensive. RADAR [6] is the first explored indoor positioning system, which computes the user location based on the RSS [7][8] from wireless local area network (WLAN). It is an attractive and suitable solution for indoor positioning since it reuses the existing and pervasive WLAN infrastructure. However, RSS-based location estimation is challenging because the radio signal is easily affected by reflection, refraction, shadowing and scattering. To resolve the problem of unstable RSSs in indoor environment, fingerprinting is considered as a feasible solution. Using the fingerprint technique with smart antenna system (SAS), which uses multi-antenna to form several logical APs with different radio coverage areas, three major processes for location estimation are required: 1) building the indoor radio map, 2) logical AP selection and 3) location estimation. The fingerprint technique consists of the offline phase and the online phase. During the offline phase, the target indoor area is logically partitioned into a number of equal subareas. For simplicity, all the corners of subareas could be arranged as the reference points (RPs) in indoor radio map. After then, a smart phone is placed on every RP to transmit a number of dummy packets to every logical AP co-located within the centric SAS. For each dummy packet, the SAS measures the RSS and stores it with the coordinate of corresponding RP and the index of logical AP into the database in designated server. During the online phase, SAS measures the RSSs of received packets sent from one mobile device at unknown position and then forwards them to designated server for further processing. Based on these received RSSs, the server calculates the possible position of mobile device by means of online location algorithm. The estimated location may either fully match or just closely match an RP in database.

Without loss of generality, the online location algorithm often relies on well-known pattern-matching algorithms including the k-nearest neighbor [1], neural network, the probabilistic approach [9], and so on. In other words, the pattern-matching algorithm tries to figure out the possible location according to the known relation between the RSS and the position of mobile device. From our observations, the weighted-based kernel function, which has been proposed in [10], to improve the indoor positioning accuracy is very suitable for SAS-based online location estimation.



(a) System block diagram (b) The vertical view of antenna array layout

Fig. 1. The architecture of smart antenna system

Recall that constructing radio map is the most critical step, where the accuracy of location estimation depends on the number of RPs. Obviously, the process of collecting RSSs often takes much time as there are many real RPs in radio map. To reduce the complexity of offline process, this paper proposes an efficient scheme, which utilizes the signal propagation model, to construct virtual RPs in radio map. Those calculated RSSs on virtual RPs are treated as the training data on real RPs. The proposed scheme not only shortens the RSS collecting time but also achieves high accuracy for indoor positioning. Similarly, using SAS to estimate indoor location also requires longer time to collect all RSS information. Combining this scheme with SAS has the advantage of easily obtaining enough RSSs information to build the radio map in a more efficient way.

The organization of this paper is described as follows. In Section II, we introduce the developed SAS for indoor location estimation. Section III is devoted to the proposed scheme of efficiently constructing the radio map and the applied indoor estimation algorithm. In Section IV, we describe the experimental environment and the obtained results. Finally, Section V concludes this paper and also gives some remarks for future works.

II. Introduction of Smart Antenna System

Smart antenna system is an emerging technique to promote the communication efficiency in wireless networks. It works by taking the advantage of the diversity effect at the transceiver in wireless systems. The diversity effect is used to decrease the error rate during data communication and to increase data transmission rate between transmitter and receiver.

The SAS designed for indoor location estimation is composed of eight directional antennas which are divided into horizontal plane (with horizontal polarization) and vertical plane (with vertical polarization). Each plane has four antennas to cover four different directions, one antenna for one direction. The SAS generates 16 (4×4) distinct antenna sets by combining any pairs of antennas from two planes, where one antenna from horizontal plane and the other antenna from vertical plane. As a result, those antenna sets have different signal coverage and trajectory to provide a better characteristic for indoor location estimation. For brevity, in this paper the antenna set is called as 'logical AP' ,which represents for the traditional AP from the aspect of WLAN client. The block diagram

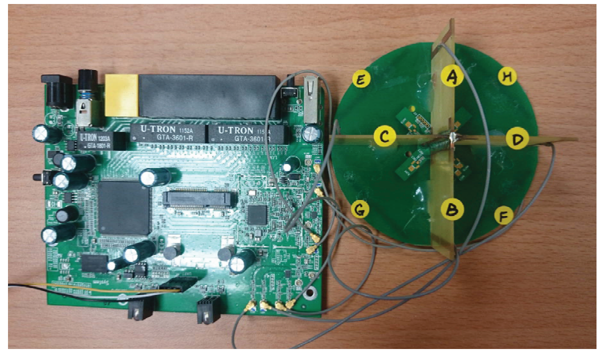


Fig. 2. Smart antenna system

TABLE I
ANTENNA SET CONFIGURATION

Antenna set No.	Antennas	Antenna set No.	Antennas
0	A:E	8	C:E
1	A:F	9	C:F
2	A:G	10	C:G
3	A:H	11	C:H
4	B:E	12	D:E
5	B:F	13	D:F
6	B:G	14	D:G
7	B:H	15	D:H

of SAS is shown in Fig. 1(a)[13]. The embedded platform includes a programmable 2×2 IEEE 802.11n Media Access Control (MAC) Controller and it uses 8 general purposes I/Os (GPIOs) to connect to two switches for dynamically selecting 2 antennas from 8 antennas (one from horizontal plane and one from vertical plane) for transmissions and receptions. The state of the switch is determined by the DC voltage. The RF output connects with the designed antenna array on the circle plate as shown in Fig. 1(b)[13]. In the antenna array, four antennas, denoted as ANT_A, ANT_B, ANT_C, and ANT_D, respectively, are formed an angle 90 degree with respect to the other four antennas, denoted as ANT_E, ANT_F, ANT_G, and ANT_H, respectively. Fig. 2[13] is the photograph of smart antenna system and Table I[13] shows the configuration between the index of antenna set and corresponding antennas controlled by MAC controller. For readability, terms 'logical AP' and 'antenna set' are interchangeable.

III. Smart Indoor Location Estimation

If the indoor signal propagation model is able to derive the precise path loss, the RSSs on any RP in indoor environment could be obtained without any actual measurement. However, from our observations, it is very difficult to use a signal propagation model with single pass loss parameter set to cover the whole indoor environment. That is why the convenient way to achieve more precise indoor positioning is to arrange as more RPs as possible regardless of the signal propagation model. However, because the SAS aims to generate distinct antenna patterns in different directions, the path loss parameters for different antenna sets would be naturally varied even in the same indoor environment. In other words, if an RP in the indoor radio map can have different path

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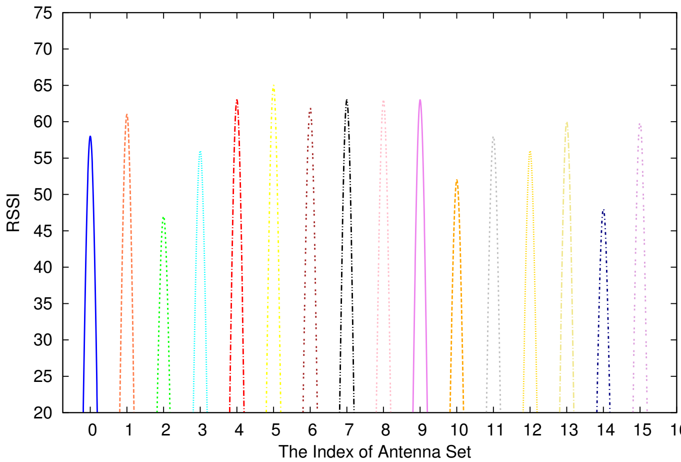

Fig. 3. The real RSS values derived from measurements

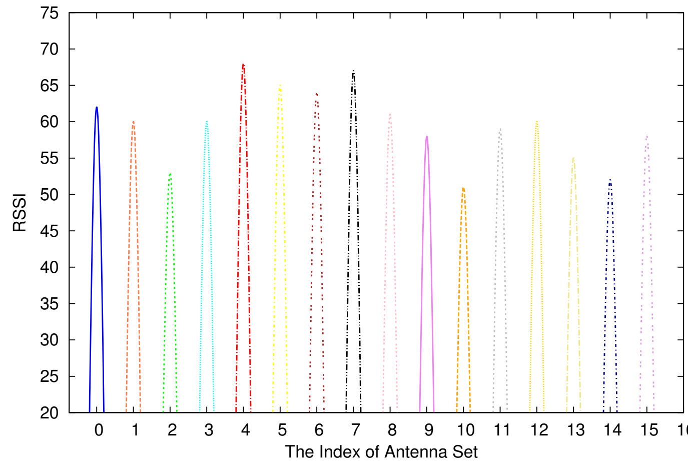

Fig. 4. The virtual RSS values generated from signal propagation model

loss parameters to reflect different antenna patterns, the indoor signal propagation model could be applicable for fingerprint-based indoor location estimation. It also implies that the number of real RPs for building indoor radio map could be reduced significantly. In a word, the way of increasing the RPs in this paper is to utilize the real measurements from real RPs to generate virtual measurements of other new RPs, called virtual RPs, in database for shortening the offline process meanwhile maintaining high positioning accuracy.

A. Location Estimation Algorithm

At the beginning, we simply assume RSS values are random variables and they could be modeled as a Gaussian distribution. The whole location process is composed of two phases. The offline phase is to build the radio map which is the key part for the fingerprint technique. As mentioned above, with the signal propagation model, we are going to further generate more virtual RPs in radio map for online computation.

We divide offline phase into two steps: 1) constructing the radio map which stores RSS values from 16 antenna sets (i.e., 16 logical APs) at every real RP and 2) performing the logical AP selection to find a proper candidate set of logical APs for online location estimation. This set could be regarded as the basis for subsequent online computation. Online phase is also composed of two steps: 1) collecting RSS values from the logical APs in candidate set at unknown location, and 2) estimating the user location based on the RSS values from logical APs in candidate set and the radio map prepared during offline phase.

B. Probabilistic-based Location Estimation

1) Scheme of Virtual Fingerprint Construction: Generally, the relation between signal strength and distance can be expressed by the following equation:

$$P(d)[dBm] = P(d_0)[dBm] + 10\gamma \log_{10}(\frac{d}{d_0}) + X_\sigma \qquad (1)$$

where $P(d_0)$ represents the transmitting power of a wireless device at the reference distance d_0, d is the distance between the wireless device and the SAS, γ is the path loss exponent and X_σ is the shadow fading which follows zero mean Gaussian distribution with σ standard deviation. To figure out d_0 and d, we should define $d_{AP}(R) = \sqrt{(X_R - X_{AP})^2 + (Y_R - Y_{AP})^2}$ as the distance between smart antenna AP and the real RP, where (X_{AP}, Y_{AP}) and (X_R, Y_R) denote the coordinates of smart antenna AP and a real RP respectively.

Owing to the diverse radio field pattern and trajectory in SAS, each antenna set should have its own distinct value of path loss parameter. For the case that the SAS is located in the center within the target area, we can easily figure out the path loss exponent (γ) for each antenna set by two real RPs and then use it to generate new RPs in the radio map. Let m denote the index of antenna set and γ_m denote the path loss exponent of the m-th antenna set. More specifically, we first carefully choose two real RPs (along one radiation direction) from the radio map and then calculate the path loss exponent of every antenna set. After deriving all the path loss exponents (i.e., the set of { γ_1, γ_2, ..., γ_{16} }), the RSSs of a new virtual RP with specified distance to SAS are derivable. Afterward, the virtual RP and corresponding RSSs are added into the radio map. Fig. 3 shows the real RSS values derived from measurements at a certain RP and Fig. 4 shows the virtual RSS values generated by the proposed virtual fingerprint construction scheme. From these two figures, it reveals that the virtual RP indeed has similar radio characteristic compared with the actual measurements.

In this paper, we let N denote as the number of total RPs for online computation and N_R and N_V respectively represent the numbers of real RPs and RPs in the database. In this paper, we let $N_R = 24$ and $N = 36$, which indicates that there are 12 virtual RPs (N_V=12) in the database.

2) The Algorithm of Antenna Set Selection: For some locations, the measured RSSs from one logical AP might be similar to the other logical AP. Such similarity will lead to biased location estimation and redundant computations. Therefore, it is important of using antenna set selective technique to determine a proper candidate set of logical APs for location estimation.

The selection methodology is to choose a set of logical APs

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with the highest correlation of RSSs. Therefore, the highest correlation of RSS could provide the highest probability of coverage over time [11]. It is worthwhile to notice that the logical APs with the strongest signal with respect to WLAN client may not provide the best positioning accuracy [12].

Here, we briefly introduce the weighted-based kernel function [10] and explain how to calculate the weighted values for the SAS. First, the information of each logical AP is quantified by calculating the signal discrimination between different RPs. Let $\bar{o}_m$ denote the mean value of RSSs obtained from the m-th antenna set. We have

$$\bar{o}_m = \frac{1}{N \cdot N_{rssi}} \sum_{i=1}^{N} \sum_{j=1}^{N_{rssi}} o_{i,m(j)}, \tag{2}$$

where N, N_{rssi}, and $o_{i,m(j)}$ represent the total number of RPs, the number of collected RSSs at every RP and the j-th ($1 \leq j \leq N_{rssi}$) collected RSS at the i-th ($1 \leq i \leq N$) RP obtained from the m-th ($1 \leq m \leq M$) logical AP, respectively. Therefore, the information of the m-th logical AP denoted as η_m is given by

$$\eta_m = \frac{1}{N \cdot N_{rssi}} \sum_{i=1}^{N} \sum_{j=1}^{N_{rssi}} (o_{i,m(j)} - \bar{o}_m). \tag{3}$$

It is noted that the RSS does not change significantly at different RPs if η_m is small. On the contrary, the higher value of η_m reveals that the RSSs varies obviously at different RPs.

To obtain a quantitative metric by calculating the spatial likelihood of the measured RSSs, a quasi entropy function denoted as $H(\cdot)$ is used to determine the weight ϖ_m. We have

$$\varpi_m = 1 + \frac{H(1-\eta_m^*)}{\max\left[H(1-\eta_m^*)\right]}, \quad \text{if } 0 < m \leq M, \tag{4}$$

where $\eta_m^* = \frac{\eta_m}{\sum_{m=1}^{M} \eta_m}$ and M indicate the normalized value and the number of logical APs respectively. The value of weight ranges from 1 to 2. The weighted value ϖ_m is related with the value η_m. [13] has shown that the order of η_m does not affect the positioning accuracy in SAS. Therefore, the weighted value ϖ_m could be ignored in SAS.

3) Online Location Estimation: This paper adopts the theorem of Bayesian Network [14] to estimate the conditional probability of each location according to the observed samples during the offline and online phases.

Let l_i and O_i denote the location of the i-th RP and the mean RSS observation set of the i-th RP respectively. The vector of RSS values for the i-th logical AP could be denoted as $O_i = [O_{i,1}, O_{i,2}, \ldots, O_{i,M}]^T$. According to the inference form [15], given the observation O, the posterior distribution indicates the likelihood of location l_i. We have

$$p(l_i|O) = \frac{p(O|l_i)p(l_i)}{p(O)} = C \cdot p(O|l_i), \tag{5}$$

where C indicates a constant value when the value of l_i follows a uniform distribution and and O is the online measured RSS. To estimate $p(O|l_i)$, the kernel-based method [9][16] could exploit the probabilistic weighting function by means of the kernel density estimator and training data. We have

Algorithm 1 Summaries of the offline and online phases

Offline phase:

1: For the i-th RP, measure RSS from M logical APs to form observation set $O_i = [O_{i,1}, O_{i,2}, \ldots, O_{i,M}]^T$.
2: Figure out the distance from an RP, say R, to the central AP
$d_{AP}(R) = \sqrt{(X_R - X_{AP})^2 + (Y_R - Y_{AP})^2}$
3: Using d,d_0 and the RSSs of two specified real RPs to derive the path loss exponents $\gamma_m (1 \leq m \leq M)$ through the propagation model:
$P(d)[dBm] = P(d_0)[dBm] + 10\gamma \log_{10}(\frac{d}{d_0}) + X_\sigma$
4: Use the propagation model with derived pass loss exponent to compute the RSSs of a virtual RP.
5: Repeat step 2 to step 5 to generate all required virtual RPs.
The number of total RPs: N
The number of real RPs: N_R
The number of virtual RPs: N_V=N-N_R
6: Estimate the mean value ($\bar{o}_m$) of Gaussian distribution in the space for the m-th logical AP.

Online phase:

7: **for** $i = 1$ to N **do**
8:     **for** $j = 1$ to N_{rssi} **do**
9:         **for** $m = 1$ to M **do**
10:             $K(O, O_i(j)) = \exp\left\{\frac{-1}{2\sigma^2} \sum_{m=1}^{M} [o_m - o_{i,m}(j)]^2\right\};$
11:         **end for**
12:         $p(O|l_i) = \frac{1}{N_{rssi}} \sum_{j=1}^{N_{rssi}} K(O, O_i(j));$
13:     **end for**
14:     $\hat{L} = \sum_{i=1}^{N} l_i p(O|l_i);$
15: **end for**

$$p(O|l_i) = \frac{1}{N_{rssi}} \sum_{j=1}^{N_{rssi}} K(O, O_i(j)), \tag{6}$$

where $O_i(j)$ represents the j-th measured fingerprint at the i-th location and the Gaussian kernel function, $K(O, O_i(j))$, is given by

$$K(O, O_i(j)) = \exp\left\{\frac{-1}{2\sigma^2} \sum_{m=1}^{M} [o_m - o_{i,m}(j)]^2\right\} \tag{7}$$

where σ is an adjustable parameter which controls the accuracy of the location estimation. In this paper, we set σ as 0.5 because the experimental shows the accuracy can reach 90% when σ = 0.5. Therefore, the possible location can be derived

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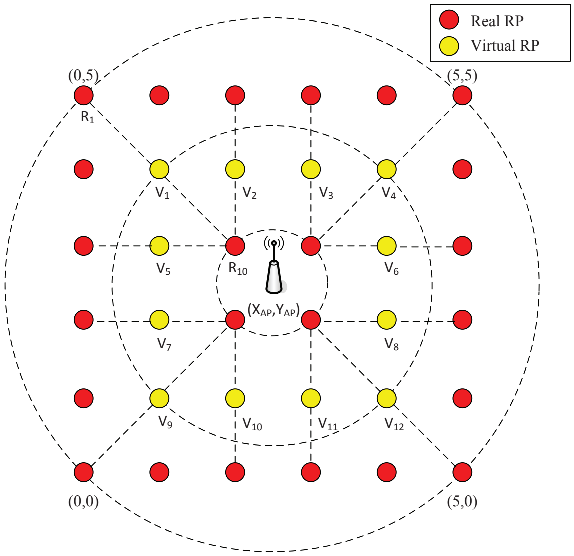

Fig. 5. The Layout of Radio Map

by

$$\hat{L} = \sum_{i=1}^{N} l_i p(O|l_i). \tag{8}$$

To wrap up, we give the summary of algorithm to clearly describe the offline and online phases.

IV. Experimental Environment and Results

A. Experimental Environment

Fig. 5 illustrates the layout of the indoor radio map, where the red nodes represent the real RPs and the yellow nodes represent the virtual RPs which are not really measured. The dash circles mean the radio wave contours generated from the SAS in the center and the dash line delegates how to use real RPs to derive the virtual RPs according to the signal propagation model. More specifically, a dash line always crosses two red nodes (i.e., real RPs) and one yellow node (i.e., virtual RP), and the yellow node just locates between two red nodes. It means that the new data of the yellow node is constructed within the range of known data of two red nodes by means of interpolation method. For applying the signal propagation equation, we need to have two distances (d and d_0) with respect to centric AP and two RSS values ($P(d)$, and $P(d_0)$) per logical AP to derive the real path loss exponent γ_m of the m logical AP. To do that, we need two real RPs for constructing every virtual RP. For the sake of explanation, we use the virtual RP, say V_1, as an example to explain how the scheme uses two real RPs, say R_1 and R_{10}, to generate virtual RP, V_1. Because the distances between AP and real RPs (R_1 and R_{10}) and the sixteen mean RSSs of sixteen logical APs measured at real RPs (R_1 and R_{10}) are known, the path loss exponent γ_m of the m-th logical AP along the path from R_{10} to R_1 is known also. By simply replacing the real RP R_1 as virtual RP V_1, the sixteen mean RSSs of sixteen logical APs probably measured at virtual RP V_1 can be derived if the sixteen path loss exponents are known already.

TABLE II
EXPERIMENTAL PARAMETERS

Number of real RPs (N_R)	24
Number of virtual RPs (N_V)	12
Number of total RPs in radio map (N)	36
Number of access points	1
Number of antennas in the AP	8
Number of logical APs (M)	16
Distance between adjacent location in x-axis	0.5 m
Distance between adjacent location in y-axis	0.5 m
Number of measured RSS data per real RP (N_{rssi})	500

The set of experimental parameters is shown in Table II. The number of real RPs (N_R) is 24 and the number of virtual RPs (N_V) in radio map is 12. These virtual RPs play the role of real RPs. Consequently, the number of total RPs N is 36.

B. Experimental Results

Fig. 6 shows the path loss exponent γ as a function of virtual RP and logical AP. It can be observed that the path loss exponents are somehow different in the considered indoor space because of the diversity effect of SAS. As a consequence, the constructed RSSs of a virtual RP are depending on the location and they are different from that of the others. Fig. 7 indicates the numerical result that depicts the accuracy (in percentage) and standard deviation of error (in meter) respectively derived from the methods with and without applying the virtual fingerprint construction scheme. For fair comparisons, the numbers of real RPs considered in the method without the virtual fingerprint construction scheme are 36 (denoted as 36 R_{RP}) and 24 (denoted as 24 R_{RP}), and the numbers of real RPs and the number of virtual RPs considered in the method with the virtual fingerprint construction scheme is 24 and 12 (denoted as 24 R_{RP} + 12 V_{RP}). It is not difficult to find that using more RPs in location estimation will result in a higher accuracy and lower standard deviation of error in Fig. 7. That is, the method using only 24 real RPs gets the lowest performance in terms of accuracy and standard deviation of error. Moreover, the performance of the method using 24 real RPs and 12 virtual RPs is very close to the method using 36 real RPs. It implies that the proposed virtual fingerprint construction scheme can perform as good as conventional method but consuming less overhead during offline phase.

Fig. 8 illustrates the positioning accuracy derived from the method with the virtual fingerprint construction scheme (24 R_{RP} + 12 V_{RP}) as a function of the number of logical APs in candidate set. The more bright areas represent more precise indoor positioning. From Fig. 8(d), we can find that, when sixteen logical APs of the SAS are included in the candidate set, the proposed scheme can obtain the best positioning accuracy level. It implies that the antenna diversity effect of developed SAS is very obvious, which is very useful for indoor location estimation.

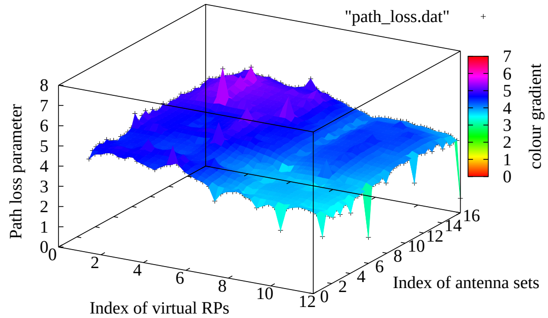

Fig. 6. Path loss exponents as a function of RP and logical AP

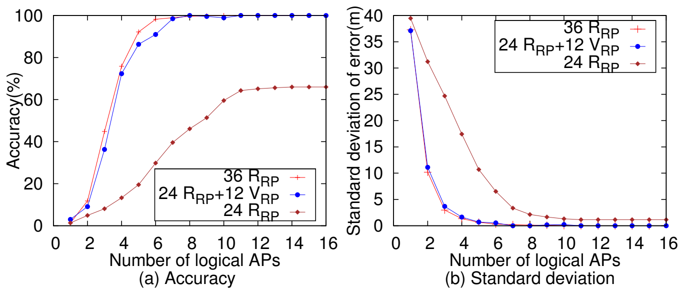

Fig. 7. Comparisons of location accuracy and standard deviation of error among original method (36 R_{RP} or 24 R_{RP}) and proposed method (24R_{RP}+12V_{RP})

V. Conclusions

To realize the fingerprint-based indoor location estimation, the offline radio map construction is definitely essential for the performance. Generally, this process requires a lot of time to gather training data from a considerable amount of reference points. In this paper, we proposed a virtual fingerprint construction scheme to shorten the offline process by means of reducing the number of real reference points in radio map. More precisely, the vanished reference points are automatically recovered by applying the signal propagation model. Experimental results show that, the smart antenna system with the proposed scheme can easily generate the valid virtual RPs which have very similar characteristic of radio signals as the measured ones. Experimental results also reveal that the proposed scheme is workable in realistic indoor environment and the virtual fingerprint construction scheme can reduce 33% (12/36) real RPs during the offline phase.

The concept behind the virtual fingerprint construction scheme is the interpolation method. Our future work is to use the extrapolation method on the virtual fingerprint construction scheme to build unlimited indoor radio map with very small amount of real reference points.

Acknowledgments

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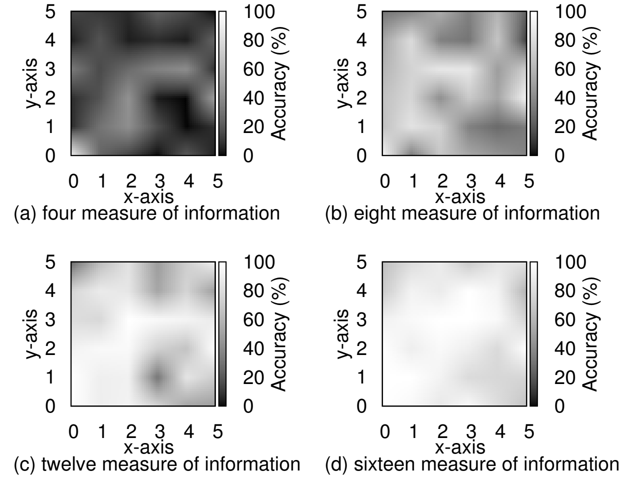

Fig. 8. The accuracy estimation under different number of antenna sets.

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