Stochastic Models of Traffic Flow Balancing and Management of Urban Transport Networks

Dmitrii Zhukov
Institute of Information Technologies
Moscow state technical university, MIREA
Moscow, Russia
e-mail: ZhukovDm@yandex.ru

Sergey Lesko
Institute of Information Technologies
Moscow state technical university, MIREA
Moscow, Russia
e-mail: Sergey@testor.ru

Anton Alyoshkin
Institute of Information Technologies
Moscow state technical university, MIREA
Moscow, Russia
e-mail: Antony@testor.ru

***Abstract*— In this paper, we developed models and algorithms for managing stochastic flows with indeterminate characteristics of distributing static parameters in urban transport networks. The proposed models describe the dependence of the single nodes blocking probability on the urban traffic parameters changing over time. In addition, we have modeled a city transport network and traffic flow with the traffic lights switching time regulated according to the proposed model and with rigidly set switching regimes (a 'conventional traffic' mode). The conducted research of the line-up lengths appearing at the traffic lights exhibits a double reduction of the traffic jams when using the elaborated model of the 'regulated' traffic lights in comparison with a conventional model of traffic management.**

Keywords-transport network; stochastic dynamics of transport network nodes blocking; traffic flow balancing; traffic flows management algorithms.

I. Introduction

In order to develop efficient algorithms of information-and-transport systems performance, one should have adequate mathematical models.

The mathematical models used to analyse the traffic networks are diverse according to the tasks they solve, mathematical tools employed and the degree of detailing the traffic flow and the data used.

Therefore, it is impossible to provide an exhaustive classification of the transport models, yet we can tentatively specify three major classes of models based on the tasks solved, i.e.:

- Forecast models. They solve the tasks of defining a number of averaged parameters of the traffic flow, e.g., the flow rate, the quantity of automobiles and passengers on various parts of a road, the transfer volume etc.
- Simulation models. They allow to describe the traffic flow dynamics, reproducing the movement of any single vehicle separately. The application of simulation models lets us estimate the traffic flow dynamics, speed, behaviour and length of line-ups and traffic jams and some other parameters.
- Optimisation models. They aim at optimising trucking or passenger travel routes, improving the calculations of traffic lights regimes, defining optimal configuration of the network model and so on.

The now existing models of traffic flow dynamics can also be classified by their properties and types [1]:

- Macroscopic models. They describe the movement of objects in averaged terms, e.g., density, vehicle cruising speed etc.
- Kinetic model. When describing a traffic flow, one equals it to the flow of some liquid, thus such models are also called hydrodynamic.
- Microscopic models describing the movement of each vehicle more precisely in comparison to the macroscopic models.

A kinetic approach to building transport models is the following. The traffic flow is described via density in the phase space, i.e., in the field of coordinates and speeds of the vehicles [1]. A kinetic equation defines the dynamics of the phase density. A kinetic approach is closer to a micro-level, whereas an averaged description allows transition to a macro-level. The main benefit of the kinetic model is that one can use it to develop macroscopic models. Knowing how phase density changes over time, we can calculate macroscopic features of the traffic flow, e.g., the average speed and the density of the traffic. Kinetic models consider the changes of automobile speeds accounted by processes of interaction. Interaction means the following: if a speedier car catches up with a slower one moving ahead, the former must either slow down or pass it. In a freely moving traffic flow, there is natural distribution of cars by 'desirable' speeds.

 ISBN: 978-1-61208-505-0

Desirable is the speed at which the car could move without any obstacles or interactions. One of the most serious drawbacks of this model is the hypothesis of an automobile chaos. According to it, during mutual interactions of cars there is no connection between their speeds.

Kerner theory [2] of the three-phase traffic flow can be attributed to macroscopic models, as it can predict and explain empirical properties of dense traffic breakdown and resulting space and time structures in the traffic flow.

One of the most efficient micro-models is the cellular automation model (Cellular automata models, CA [3]). Cellular automata are idealised representations of physical systems in which the time and space are represented as discreet, and all elements of the system have some discreet range of possible states.

A road is divided into conditional cells of the same length Δx, at that at each moment the cell is either empty or occupied by a single vehicle. At each time step t → t + 1, the condition of all cells simultaneously (in a parallel way) updates according to some set of rules. The choice of some set of rules defines the diversity of CA options [4]. Traffic models based on CA can correlate the traffic flow dynamics at a micro-level with the traffic flow behaviour at a macro-level.

We should note that the afore-mentioned models are not universal and have some flaws necessitating the search for new models, e.g., the ones founded on stochastic dynamics.

The rest of the paper is structured as follows. In Section II, we will describe stochastic model of traffic flow. In Section III, we will describe the results of mathematical simulation of the traffic flows in urban transport networks and improvements of road situations when we use the model of 'managed' traffic lights. We conclude in Section IV.

II. A STOCHASTIC MODEL OF TRAFFIC FLOW DISTRIBUTION WITH INDETERMINATE CHARACTERISTICS IN URBAN TRANSPORT NETWORKS

To develop a dynamic model of traffic network performance, we propose decomposing the task set and dividing it into solutions of two levels:

- The level of describing a single node performance dynamics.
- The level taking into consideration the network topology and a single node performance dynamics.

The essence of the model we developed for performance of single nodes is the following. If we consider the change of traffic flow as a random process and we set for each direction, each node of the transport network (a junction) a critically admissible number of cars in line $L_{i,j}$, then we can define the probability $P(L_{i,j}, t)$ of the situation that by moment t the number of cars in a line will not exceed $L_{i,j}$ (there is no traffic jam).

Suppose that over some time interval τ there arrive ε cars and leave ξ cars in the i direction of the line at j junction. The whole data processing will be adding single steps h having duration τ, with $\dfrac{\varepsilon}{\tau} = \lambda$ being the intensity of the traffic inflow, and $\dfrac{\xi}{\tau} = \mu$ being the intensity of traffic outflow.

Let us denote by $P_{x-\varepsilon,h}$ the probability of the number of cars $(x\text{-}\varepsilon)$ in line after h steps of processing, by $P_{x,h}$ the probability of x cars, and by $P_{x+\xi,h}$ the probability of $(x+\xi)$ cars. Then probability $P_{x,h+1}$ (see Figure 1) of x cars at step $h+1$ will equal:

$$P_{x,h+1} = P_{x-\varepsilon,h} + P_{x+\xi,h} - P_{x,h}$$

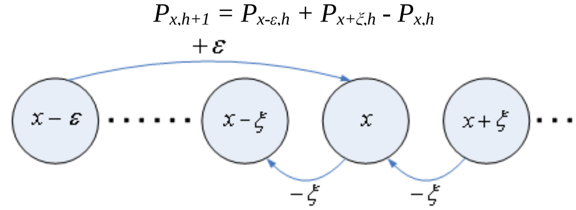

Figure 1. The scheme of probable transitions between the states characterising the number of cars at j junction in i direction at h+1 step of traffic lights performance.

Let us introduce $t=h\tau$, where t is the total time of processing, we thus get:

$$P(x,t+\tau)=P(x-\varepsilon,t)+P(x+\xi,t)-P(x,t)$$

Expanding this equation in a Taylor series, we obtain:

$$P(x,t)+\tau\frac{dP(x,t)}{dt}+\frac{\tau^2}{2}\frac{d^2P(x,t)}{dt^2}+\ldots=$$
$$=P(x,t)-\varepsilon\frac{dP(x,t)}{dx}+\frac{\varepsilon^2}{2}\frac{d^2P(x,t)}{dx^2}-\ldots+$$
$$+P(x,t)+\xi\frac{dP(x,t)}{dx}+\frac{\xi^2}{2}\frac{d^2P(x,t)}{dx^2}+\ldots-P(x,t).$$

The second derivative of t can be excluded as it naturally describes the process at which cars themselves could bear additional cars. Taking into account the members in the left part containing no more than the first derivative of t, and the members of the right part having no more than the second derivative of x, we receive:

$$\tau\frac{dP(x,t)}{dt}=\frac{\varepsilon^2+\xi^2}{2}\frac{d^2P(x,t)}{dx^2}-(\varepsilon-\xi)\frac{dP(x,t)}{dx},$$
$$\frac{dP(x,t)}{dt}=\frac{\lambda^2+\mu^2}{2\mu}\frac{d^2P(x,t)}{dx^2}-(\lambda-\mu)\frac{dP(x,t)}{dx}.$$

Assuming that μ and λ do not depend on x and introducing the insymbol $a=\dfrac{\lambda^2+\mu^2}{2\mu}$ and $b=\lambda-\mu$, we get:

$$\frac{dP(x,t)}{dt}=a\frac{d^2P(x,t)}{dx^2}-b\frac{dP(x,t)}{dx}.$$

As the function $P(x,t)$ is continuous, we can transgress from probability $P(x,t)$ to the probability density $\rho(x,t)$, which enables us to formulate and solve a boundary problem for describing a stochastic model of processing applications at a single node with indeterminate parameters of the statistical law of their incoming times distribution.

With the number of cars $x=L$ lining up for j junction in i direction, where L is some critical limit number, we assume that the node of processing (j junction in i direction) becomes overloaded and a traffic jam is formed. The probability of detecting such a state will be different from zero, and the probability density defining the traffic flow in state $x=L$ should be put down as 0 (we are trying to avoid this condition), i.e.:

$$\rho(x,t)_{x=L}=0 \tag{a}$$

We choose the second limit condition based on the fact that condition $x=0$ defines a standstill in the processing. The probability to detect such a state will be nonzero, however, the probability density defining the traffic flow under condition $x=0$ must be laid down as equal to 0 as well. We must also strive to avoid this condition as it correlates to the case when the traffic lights do not close this direction, and it contradicts the logic of its work, i.e.:

$$\rho(x,t)_{x=0}=0 \tag{b}$$

As at time $t=0$ (the start of the calculation) there may be x_0 cars under the processing. Let us set the initial condition in the following form:

$$\rho(x,t=0)=\delta(x-x_0)=\begin{cases}1, x=x_0\\ 0, x\neq x_0\end{cases}$$

Since the initial condition is set as a δ-function, it leads to the fact that the solution of the obtained differential equation remains indiscreet (continuous) at point $x=x_0$ and the equation will suffer derivative discontinuity at this point.

Employing operational calculus methods for probability $P(L_{i,j}, x_0|t)$ of the traffic jam not being formed by some t moment (the number of cars in a line will not exceed $L_{i,j}$), we can obtain the following expression:

$$P(L_{i,j},x_0\,|\,t)=2e^{-\frac{2b_{i,j}x_0+b_{i,j}^2t}{4a_{i,j}}}\cdot$$

$$\cdot\sum_{n=1}^{M}\frac{e^{\frac{b_{i,j}L_{i,j}}{2a_{i,j}}}\sin(\pi n\frac{x_0}{L_{i,j}})+\sin(\pi n\frac{L_{i,j}-x_0}{L_{i,j}})}{(-1)^{n+1}\left\{\pi n+\frac{b_{i,j}^2L_{i,j}^2}{4\pi n a_{i,j}^2}\right\}}\cdot e^{-\frac{\pi^2n^2a_{i,j}t}{L_{i,j}^2}} \tag{1}$$

where $a_{i,j}=\frac{\mu_{i,j}^2+\lambda_{i,j}^2}{2\lambda_{i,j}}$ and $b_{i,j}=\lambda_{i,j}-\mu_{i,j}$, $\mu_{i,j}$ are the number of cars leaving j-node of traffic network (junction/traffic lights) in i-direction per unit of time (an outflow), $\lambda_{i,j}$ is the number of cars entering the node per unit of time (an inflow), t is time, x_0 is the number of cars in a line at the start time of traffic lights operation step.

Solving (1) relative to time t allows determining optimal time intervals of traffic lights switch on. Yet it is input-intensive computational task. Taking into consideration that the computations should be simultaneously carried out for multiple directions and junctions, and one should also synchronise (see (2)) the traffic inflows and outflows as neighbouring junctions, it is reasonable to use parallel computations to simulate the traffic flow.

$$x_{0i,j}^{k}=$$

$$x_{0i,j}^{k-1}+\frac{1}{r}\sum_{i-1}^{r}(\mu_{i,j}^{k-1}\tau_{i,j}^{k-1}+\Delta\lambda_{i,j}^{k-1}T_{i,j}^{k-1})-\mu_{i,j}^{k}t_{i,j}^{k} \tag{2}$$

$$\lambda_{i,j}^{k}=\frac{x_{0i,j}^{k}V_{D1}}{l_{i,j}},$$

where $x_{0i,j}^{k-1}$ is the number of cars which did not manage to get through in direction i of j junction after performing *(k-1)* step, r is the number of directions at the junction, $\mu_{i,j}^{k-1}$ are the outflows at step *(k-1)* per each r direction at the chosen junction. Any car from those traffic inflows at step *(k-1)* can equally likely choose at next k step one of r directions, that is why we introduce a numerical coefficient (1 / r) before the summation symbol. $T_{i,j}^{k-1}$ is the time during which the chosen direction was closed by the traffic lights (it is not the 'open' time, but the time of 'idle cycle') between two chained openings. Let us note that opening of all directions at the chosen junction may happen not in the rigidly set periodical sequence. The order of directions openings may vary depending on the traffic behavior. The time interval between two chained openings of the same chosen direction will simply be the 'idle cycle', which value $T_{i,j}^{k-1}$ can change dynamically. $\Delta\lambda_{i,j}^{k}$ is the change of traffic inflow in the chosen direction at the chosen node of traffic flow over time $T_{i,j}^{k-1}$. The total number of cars in the transport network at any time of the day corresponds to the function of the number of cars on the time of the day. $\tau_{i,j}^{k-1}$ is the time during which at *(k-1)* step the directions of traffic inflows are open while the chosen outflowing direction is closed during time $T_{i,j}^{k-1}$. $\mu_{i,j}^{k-1}$ is the traffic outflow in the chosen direction at step k, $t_{i,j}^{k}$ is the time interval of traffic lights switch on at step k of the chosen direction. It is important to determine the time interval value in order to solve the equation defining probability $P(L_{i,j}, x_0|t)$ of the fact that by time t the number of cars in line will not exceed $L_{i,j}$ (the gridlock do not form). V_{D1} is the recommended speed.

III. Simulating and Balancing the Traffic Flows in Urban Transport Networks on the Basis of the Stochastic Model

To simulate transport network and determine whether it is possible, as a matter of principle, to dynamically change the time intervals of traffic lights switching in a city to prevent traffic gridlock, besides (1) and (2), we should have a model of how the number of cars varies with the time of the day. For modelling, the total number of cars in a transport network may be set, for example, by the function depicted in Figure 2 (The traffic load is measured with 10

 ISBN: 978-1-61208-505-0

generating units (points) and correlated to the situation when all vehicles registered in Moscow or Moscow Region are on the road at the same time. According to statistics, autumn is the busiest time of the year).

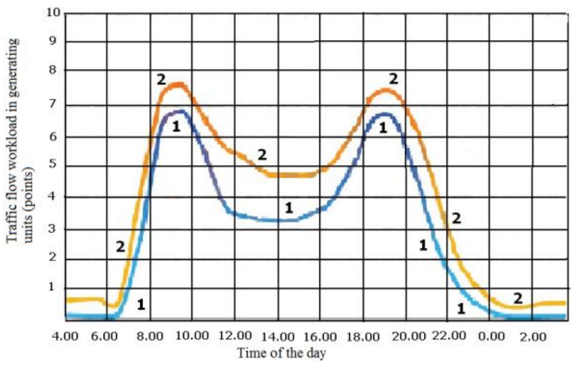

Figure 2. Traffic load in Moscow during a workday (curve 1 represents autumn of 2013; curve 2 is for autumn 2014).

Basing on (1) and (2) as well as on the function in Figure 2, we developed an algorithm of managing the transport network and tested the proposed approach.

For the simulation purposes, we elaborated a number of algorithms and software allowing us to model a city transport network and traffic situations with 'controlled' traffic lights, which are regulated according to the proposed model, and 'uncontrolled' traffic lights which have rigidly set modes of switching (classical traffic).

To serve as a technological concept, we realised the function of maps downloading as Open Street Map (OSM) and wrote a parser of this format, the output of which is a graph of transport network with featured properties of arcs (a road) and peaks (a junction). This is important for simulation and emulation of traffic flow, and the set of WPF (Windows Presentation Foundation) objects is vital for their onto mapping.

At the second stage, we constructed a structural model of a city, which featured all classes of roads, junctions, traffic directions, traffic lights and their modes, as well as cars and line-ups. Besides, we realised the tools for entering the cars according to their per diem spread (see Figure 2) and the tools to set the cars behaviour.

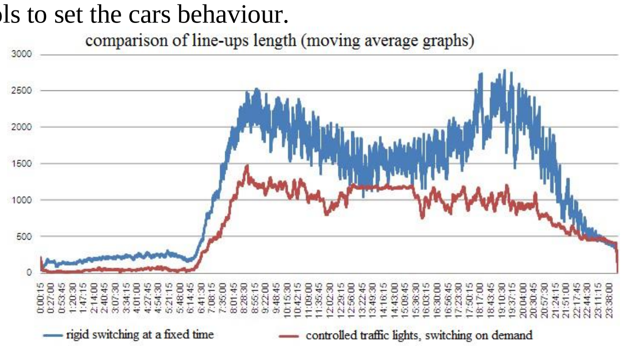

Figure 3. Comparison between performing efficiencies of the proposed model and the traffic lights switching at a fixed time

We chose the index of total length of all line-ups at all junctions of the loaded map to be the simplest efficiency criterion. A traffic jam is a line-up of cars at the traffic lights waiting for the permissive signal.

For the experiment, we downloaded a small real map of Vasilievskiy Island in Saint Petersburg and simulated daily traffic flow. Careful examination of line-ups length (see Figure 3) demonstrates that the number of gridlocks halved when we use the model of 'managed' traffic lights in comparison with the rigid classic traffic mode.

The traffic lights with fixed phases of operation help to solve the problem of regulating the traffic, yet they are less efficient and cannot react to traffic situation changes as they lack feedback. For instance, if cyclical (daily) variations could be accounted for when we develop the traffic lights operational phases, plenty of various accidental (random) factors, such as weather conditions, road works, accidents, cannot be fully factored in.

It is evident that real traffic situation can greatly diverge with the experimental mathematical simulation, but the obtained results allow us to state the adequacy of the proposed model and its potential application in designing control services in automated systems of traffic flow.

IV. CONCLUSIONS

We developed models and algorithms of controlling stochastic flows with indeterminate characteristics of statistical parameters spread in urban transport networks; they allow to describe the dependence of single node blocking probability on the traffic flow parameters over time.

In the elaborated mathematical models, we provided a description of how to service junctions (the times of traffic lights switching), accounted for the traffic burden balance of cars in the system and connection of traffic flows between neighbouring junctions. The proposed model enables the creation of a dynamic model using a real transport network map and thus emulating its work and testing the way gridlocks are formed.

Simulation of urban transport network and traffic situation according to the proposed model of 'controlled' switching times of traffic lights in comparison with fixed modes of switching ('classical traffic') shows a double reduction of gridlocks when we use the former model of 'controlled' switching instead of the latter one.

In our future works we plan to develop traffic lights control algorithms based of stochastic models of balancing and flow control that focus not on the "priority of the shortest possible route", but on the load state of each node (or group of nodes). Given the fact that some nodes can be soon overloaded and such nodes should be balanced. Depending on how close node is to critical state in a transport network, the metric for this node can be proportionally increased linearly or nonlinearly (the closer to critical state, the greater value of the metric).

We also note that the above approach can also be applied in case when the transport network is divided into several areas and inner traffic lights within areas have information about the topology of the other parts of the network.

ACKNOWLEDGMENT

This work is supported by Russian Foundation for Basic Research (grant 16-37-00373 mol_a).

REFERENCES

[1] S. Göttlich and A. Klar, "Model hierarchies and optimization for dynamic flows on networks," Modeling and optimization of flows on networks, Cetaro (CS), June 15–19, 2009, C.I.M.E, Courses, 2009. http://php.math.unifi.it/users/cime/Courses/2009/01/200914-Notes.pdf [retrieved: 09, 2016]

[2] B. S. Kerner, "Introduction to Modern Traffic Flow Theory and Control," Berlin: Springer, 2009.

[3] S. Maerivoet and B. De Moor, "Cellular automata models of road traffic," Physics Reports 2005, V. 419, № 1, P. 1–64., 2005.

[4] H. J. Ruskin and R. Wang, "Modeling Traffic Flow at an Urban Unsignalized Intersection," Lecture Notes in Computer Science. 2002. T. 2329. P. 0381.