ICIW 2022

The Eighteenth International Conference on Internet and Web Applications and Services

ISBN: 978-1-61208-957-7

June 26th –30th, 2022

Porto, Portugal

ICIW 2022 Editors

Pascal Lorenz, University of Haute Alsace, France

ICIW 2022

Forward

The Seventeenth International Conference on Internet and Web Applications and Services (ICIW 2022), held between June 26th and June 30th, 2022, continued a series of events that covered the complementary aspects related to designing and deploying of applications based on IP&Web techniques and mechanisms.

Internet and Web-based technologies led to new frameworks, languages, mechanisms and protocols for Web applications design and development. Interaction between web-based applications and classical applications requires special interfaces and exposes various performance parameters.

Web Services and applications are supported by a myriad of platforms, technologies, and mechanisms for syntax (mostly XML-based) and semantics (Ontology, Semantic Web). Special Web Services based applications such as e-Commerce, e-Business, P2P, multimedia, and GRID enterprise-related, allow design flexibility and easy to develop new services. The challenges consist of service discovery, announcing, monitoring and management; on the other hand, trust, security, performance, and scalability are desirable metrics under exploration when designing such applications.

Entertainment systems became one of the most business-oriented and challenging area of distributed real-time software applications and special devices industry. Developing entertainment systems and applications for a unique user or multiple users requires special platforms and network capabilities.

Particular traffic, QoS/SLA, reliability and high availability are some of the desired features of such systems. Real-time access raises problems of user identity, customized access, and navigation. Particular services such interactive television, car/train/flight games, music and system distribution, and sport entertainment led to ubiquitous systems. These systems use mobile, wearable devices, and wireless technologies.

Interactive game applications require particular methodologies, frameworks, platforms, tools and languages. State-of-the-art games today can embody the most sophisticated technology and the most fully developed applications of programming capabilities available in the public domain.

The impact on millions of users via the proliferation of peer-to-peer (P2P) file sharing networks such as eDonkey, Kazaa and Gnutella was rapidly increasing and seriously influencing business models (online services, cost control) and user behavior (download profile). An important fraction of the Internet traffic belongs to P2P applications.

P2P applications run in the background of user’s PCs and enable individual users to act as downloaders, uploaders, file servers, etc. Designing and implementing P2P applications raise particular requirements. On the one hand, there are aspects of programming, data handling, and intensive computing applications; on the other hand, there are problems of special protocol features and networking, fault tolerance, quality of service, and application adaptability.

Additionally, P2P systems require special attention from the security point of view. Trust, reputation, copyrights, and intellectual property are also relevant for P2P applications. On-line communications frameworks and mechanisms allow distribute the workload, share business process, and handle complex partner profiles. This requires protocols supporting interactivity and real-time metrics.

Collaborative systems based on online communications support collaborative groups and are based on the theory and formalisms for group interactions. Group synergy in cooperative networks includes online gambling, gaming, and children’s groups, and at a larger scale, B2B and B2P cooperation.

Collaborative systems allow social networks to exist; within groups and between groups there are problems of privacy, identity, anonymity, trust, and confidentiality. Additionally, conflict, delegation, group selection, and communications costs in collaborative groups have to be monitored and managed. Building online social networks requires mechanism on popularity context, persuasion, as well as technologies, techniques, and platforms to support all these paradigms.

Also, the age of information and communication has revolutionized the way companies do business, especially in providing competitive and innovative services. Business processes not only integrates departments and subsidiaries of enterprises but also are extended across organizations and to interact with governments. On the other hand, wireless technologies and peer-to-peer networks enable ubiquitous access to services and information systems with scalability. This results in the removal of barriers of market expansion and new business opportunities as well as threats. In this new globalized and ubiquitous environment, it is of increasing importance to consider legal and social aspects in business activities and information systems that will provide some level of certainty. There is a broad spectrum of vertical domains where legal and social issues influence the design and development of information systems, such as web personalization and protection of users privacy in service provision, intellectual property rights protection when designing and implementing virtual works and multiplayer digital games, copyright protection in collaborative environments, automation of contracting and contract monitoring on the web, protection of privacy in location-based computing, etc.

We take here the opportunity to warmly thank all the members of the ICIW 2022 technical program committee, as well as all the reviewers. The creation of such a high-quality conference program would not have been possible without their involvement. We also kindly thank all the authors who dedicated much of their time and effort to contribute to ICIW 2022. We truly believe that, thanks to all these efforts, the final conference program consisted of top-quality contributions. We also thank the members of the ICIW 2022 organizing committee for their help in handling the logistics of this event.

We hope that ICIW 2022 was a successful international forum for the exchange of ideas and results between academia and industry and for the promotion of progress in the field of Internet and Web Applications and Services.

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Ontology for the Context of E-Mobility: Charging Station Recommendation based on the EV Trip

Dimeth Nouicer
LaRINa, ENSTAB, University of Carthage
ENSIT, University of Tunis
Nabeul, Tunisia
dimethnoucer@gmail.com

Ikbal Chammakhi Msadaa
LaRINa, ENSTAB
University of Carthage
Tunis, Tunisia
ikbal.msadaa@ensta.u-carthage.tn

Khaled Grayaa
LaRINa, ENSTAB
University of Carthage
Tunis, Tunisia
khaled.grayaa@ensta.u-carthage.tn

***Abstract*—Electromobility (E-Mobility) is a disruptive technology that would facilitate the transition from highly polluting transport systems to carbon neutral mobility. However, for a mass adoption of this technology, several barriers need to be addressed. In this paper, we propose an ontology intended to cover a large variety of applications related to E-Mobility, ensure the semantic interoperability between the different stakeholders of the electromobility ecosystem, and ease their collaboration. We have made the choice in this paper to validate our ontology through a trip planning application as a start. This use case is particularly relevant as it helps the Electric Vehicles (EVs) drivers plan their journeys and overcome the hurdle of range anxiety in their minds by offering them the possibility to choose and book in advance the most appropriate charging points for recharging their EVs batteries. The proposed ontology is developed under Protégé framework using the Web Ontology Language (OWL) and validated through SPARQL queries with GraphDB. It is based on Smart Applications REFerence (SAREF) ontology adopted by European Telecommunication Standardization Institute (ETSI) to ensure semantic interoperability in the Internet of Things (IoT) domain.**

***Index Terms*—e-mobility; electric vehicles; trip planning; semantic web; ontology.**

I. Introduction

Electromobility has emerged as one of the most promising technologies that would facilitate the transition from highly polluting transport systems to zero-emission systems particularly when powered by Renewable Energies (RE). Indeed, as part of Paris agreement, the 197 signatory countries are committed to reduce their GreenHouse Gas (GHG) emissions and become carbon-neutral by 2050. Nevertheless, substantial technical, socio-economic, and regulatory barriers must be addressed to achieve the widespread adoption of EVs. As highlighted in [1], one of the key barriers to mass EVs adoption is "the informational barrier" which mainly refers to the range anxiety in the minds of EV drivers. Lower driving ranges, long charging times, and the need for charging infrastructure have been identified (in addition to the EV cost) as the main key factors that are slowing down a widespread use of EVs. To address this issue, we believe that providing services and applications such as trip planning and charging stations booking would help overcome the hurdle of range anxiety and encourage consumers to adopt EVs as alternative to the conventional internal combustion engine vehicles.

To reach the above goal, it is necessarily to share a common understanding of the information structure among the main stakeholders and domain-related software agents [2]. In this context, ontologies emerge as a powerful tool to set agreed-upon definitions that capture the main concepts of a domain knowledge and enable its reuse. For instance, ontologies will ease the complexity of the E-Mobility ecosystem, e.g., heterogeneity, multidisciplinary aspect and the multitude of systems and parties involved, and enable the collaboration and interoperability between the different stakeholders, e.g., EV owners, Charging Stations (CS), grid suppliers, payment and third-party services providers. Thus, ontologies will not only profit the electromobility services development such as trip planning and Charging Points (CPs) booking (which are the main use cases considered in our paper), but also to facilitate the mass adoption of other disruptive transport technologies such as Mobility as a Service (MaaS) applications and Connected and Autonomous Electric Vehicles (CAEV) [3]. MaaS is an innovative mobility paradigm that offers to consumers the possibility to get from A to B, using different transportation modes in a flexible and seamless way, through a single interface [4]. It relies on a digital platform that integrates services allowing the booking and payment of services across public and private transportation systems in a flexible, on-demand and seamless way.

The remainder of this paper is organised as follows. Section II gives a brief literature review about the use of ontologies in the transportation domain in general with a particular focus on works related to E-Mobility and eMaaS services. Section III provides an overview of the E-Mobility domain through the identification of its key stakeholders and the main technical terms that need to be introduced. Section IV is dedicated to the description of the main use case addressed by our ontology, namely the trip planning and CPs booking applications. After explaining in Section V the adopted methodology used in developing our ontology, we describe in Section VI the proposed ontology. Several usage scenarios are proposed in this section to showcase the use of the ontology in the trip planning context. The results of the ontology evaluation via the OOPS! scanner are reported in the same section. Section VII concludes the paper and sheds light on the possible future directions.

II. Related Work

The complexity of the transportation domain in general and the E-Mobility in particular has triggered the need to develop ontologies that would represent the related concepts and the relations between them in a formal and explicit way, thus facilitating the domain knowledge sharing and reuse. After explaining the methodology of developing an ontology, Yazdizadeh and Farooq [5] survey the main existing ontologies related to the smart mobility domain. They classify these ontologies into two main categories: (i) foundation ontologies related to domains such as weather, trip, etc., and (ii) transportation physical networks ontologies where we find for instance pedestrian network, cycling network, and railway network ontologies. As future directions, the authors identify MaaS as one of the most important and disruptive mobility technologies for which no ontology has been proposed yet. Garcia *et al.* [4] introduce the concept of eMaaS where E-Mobility Systems (EMS) are combined with the MaaS concept with a particular focus on Shared Electric Mobility Services (SEMS). The authors propose, through a combination of these three modules, an eMaaS system architecture with integrated smart services such as seamless trip planning and booking, payment services, fleet management and monitoring, etc. The authors however do not refer to the format of the exchanged data neither to the mechanisms that ensure its interoperability. [6] and [7] are among the rare works that have proposed an ontology that considers EVs. Nevertheless, the ontology remains generic in the sens that it addresses green energy in general and uses EVs as an example of ”devices” that consume energy. Households are also addressed in this work as consumers or producers of green energy. Moreover, the whole ontology has been built from a Business Model (BM) perspective which objective is to ensure an automated reasoning for evaluating BMs and green behaviours. Scrocca *et al.* developed an interesting ontology called ”Urban IoT ontologies” [8], [9] (UIOT) to solve interoperability issues especially for Milan Municipality. Their goal was to harmonise the data flows between service suppliers and the city of Milan. Their solution consists of a core ontology and two extensions: one for sharing mobility and one for e-mobility. Due to the specific needs of Milan Municipality, the ontology proposed is dedicated to serve the service suppliers and the government. There is no mention of what the EV drivers need or how to assist them in planning their journeys which we consider as one of the most important issues to tackle in e-mobility. Damaj et al. [3] consider the electromobility domain from a QoE perspective. They survey the quality indicators in Connected and Autonomous Electric Vehicles domain and propose a rich taxonomy that would facilitate the development of QoE-centric systems for the CAEV context.

Despite the increasing popularity of E-Mobility, current research works on ontologies for this domain remain either too generic by dealing with ontologies addressing the transportation science [5] or the smart city context [10] in general, or they tend to show EVs only as an example of a green device [6], [7]. In this paper, we propose an ontology that captures the main concepts related to electromobility and integrates the key technical features that have often been overlooked in previous works. We adopt in our ontology design a user-centric approach where the whole reasoning is made in a way to facilitate the development of services such as the trip planning service or the Charging Points (CP) booking applications, thus addressing the needs of the EV driver. Moreover, and to ensure high interoperability for the proposed ontology, we rely on the Smart Applications REFerence ontology (SAREF) adopted by the European Telecommunications Standard Institute (ETSI) as standard framework for smart applications. The core ontology is complemented by a set of extensions that are particularly relevant is our context, such as SAREF4CITY the SAREF extension for smart cities, SAREF4SYST, the extension for systems, connections and connection points and mainly SAREF4AUTO, the extension for the automotive domain. While being still under development, the latter, combined with the core ontology and other extensions, represent a solid starting point for our ontology as it has been validated by a standardization body and is candidate to be the core ontology in smart transportation domain.

III. E-Mobility Background

Before tackling the ontology development part, it is important to have a common understanding of the key aspects and challenges related to E-Mobility. First, we highlight the main actors involved in the E-Mobility ecosystem and then we define the technical terms and explain the technical features that would influence our ontology design.

A. E-Mobility Stakeholders

One of the main challenges when dealing with E-Mobility is the heterogeneity and complexity of its ecosystem. The involved parties in this domain include EV drivers, charging points operators, grid and renewable energies suppliers, and other actors such as the standardization and regulation bodies.

Other stakeholders like the policy makers, i.e., government and local authorities, are also part of this ecosystem as they play a major role in setting the E-Mobility strategy and services. EV manufacturers, payment operators and other 3rd party and maintenance service providers are also needed to assist EV drivers and EV fleet managers through their services. Note that what makes this ecosystem even more complex is that these stakeholders usually have conflicting interests and different QoE expectations. EV drivers, for instance, are motivated by finding available CPs and 3rd party service providers, decreasing the time of charge, and reducing its cost. The grid providers however, are more concerned by the stability of the grid and the increase of their profit.

B. E-Mobility Technical Terms

Before diving into the ontology development, some E-Mobility technical terms should be defined. In IEC 61851 [11], the international standard for electric vehicle conductive charging systems, 4 modes of charging are defined. Modes 1, 2, and 3 are based on Alternative Current (AC) while mode 4 corresponds to the case where the EV supply Equipment (EVSE) at the charging point delivers Direct Current (DC). In terms of speed, only mode 4 (DC) is classified as fast to ultra-fast charging. In the other modes, power is delivered at slow to normal speed.

Depending on its model, an EV has at least one AC charging inlet port to plug the vehicle to a power supply with optionally a second DC port for fast charging or even a single port for both AC and DC charging. From the EVSE side, there exists several types of connectors to plug the charging cable to the vehicle port. The type depends not only on the charging mode, but also on the car model and the country of use. For example, type 1 connector (SAE J1772) is an AC connector. While being mainly used in USA and Japan, it is also accepted in Europe. Therefore, it is crucial to ensure the compatibility between the EV port and the connector of the charging cable. In the rest of the paper and for the sake of simplicity, we refer to both the EV inlet port and the EVSE connector as ”connector”.

The charging time, which is one of the main concerns of EV drivers, can be roughly defined as the ratio between the EV battery capacity (in kWh) and the charging power (in kW). The latter is limited to the power that the CP can deliver and that the EV can accept.

To compensate the energy losses in the different equipment, an augmentation of at least 20% is necessary to reach a better approximation of the charging time.

To give an example, if we consider an EV with a battery capacity of 44.5 kWh and on-board charger of 6.6 kW, the charging time is ˜8 hours with an 11 kW AC charging point ((44.5/6.6)*1.2) as it is limited by the onboard AC charger rate. However, with an ultra-fast charging station of 250 kW, the charging time goes down to ˜13 min ((44.5/250)*1.2). The last technical property we need to introduce at this level is the State of Charge (SoC) of the EV battery. It is usually expressed as a percentage and it corresponds to the ratio between the battery remaining energy and the total capacity of the battery.

IV. A Trip Planning Application for EV drivers

The ontology we propose in this work is intended to cover a large variety of applications related to E-Mobility, ensure the semantic interoperability between the different stakeholders, and ease their collaboration when sharing knowledge. Nevertheless, we have made the choice in this paper to validate it through a trip planning application as a start. This use case is particularly important as it helps the EV drivers plan their journeys and overcome the hurdle of range anxiety in their minds by offering them the possibility to choose and book in advance the most appropriate CPs to be used for recharging their EVs’ batteries.

Figure 1 shows the input and output parameters of such an application and the decision criteria based on which the most appropriate CPs are recommended. As the figure suggests, before starting his/her journey, the EV driver enters the source and destination positions of his/her trip, the estimated departure time and the targeted SoC at arrival. We suppose that the current SoC of the EV battery can be automatically retrieved by the application via a sensor placed at the EV battery. Based on the input, the application checks one by one the following criteria:

1) Availability: This step corresponds to identifying the CPs that belong to the geographical area matching the EV driver’s path. We only list the CPs that are within the time slots of the driver’s estimated time of arrival.
2) Compatibility: Once the available CPs are identified, only those compatible with the EV are considered eligible (in terms of power, connector, etc.), as shown in Figure 2.
3) Charging Time: In addition to the availability per time slot, the EV driver has access to the estimated charging time that would enable him/her to reach the targeted SoC (cf. Figure 2.b). As explained in Section III-B, several parameters are considered to estimate this duration.
4) Cost: The EV driver can check the applied cost rates at a specific CP within a given time interval. Note that several pricing models can be implemented at this level to avoid the grid overloading at peak time or to encourage EV drivers to take alternative paths, etc.
5) Carbon Footprint: The last criteria we consider to recommend the most appropriate CP, is how ”green” is a given CP. The latter is considered green if it is supplied by renewable energy sources (solar power, wind power, etc.).

Obviously, it is possible to give access to other information that might influence the EV driver decision such as the offered facilities at a given CP, e.g., cafe, restaurant, shopping center, etc., as suggested in Figure 2.d. All these criteria might also be coupled with the driver preferences and ranked accordingly. Based on all the provided information: availability, cost, charging time, etc., the EV driver can make his/her choice and book the CP that fits the most his/her plan.

During the recommendation phase, the application identifies the area within which the EV SoC remains above a certain threshold, e.g., 20%. The zone is delimited based on the initial SoC, the consumption rate of the EV and the distance from the departure point where charging becomes inevitable and urgent beyond this area. Therefore, it would be recommended to the driver to pick a CP preferably within the that area to avoid reaching values of SoC that would harm the EV battery and impact its lifetime.

The integration of the EV driver preferences and coupling

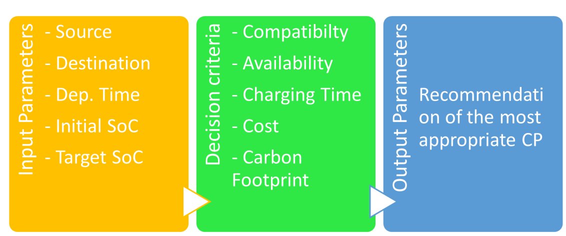
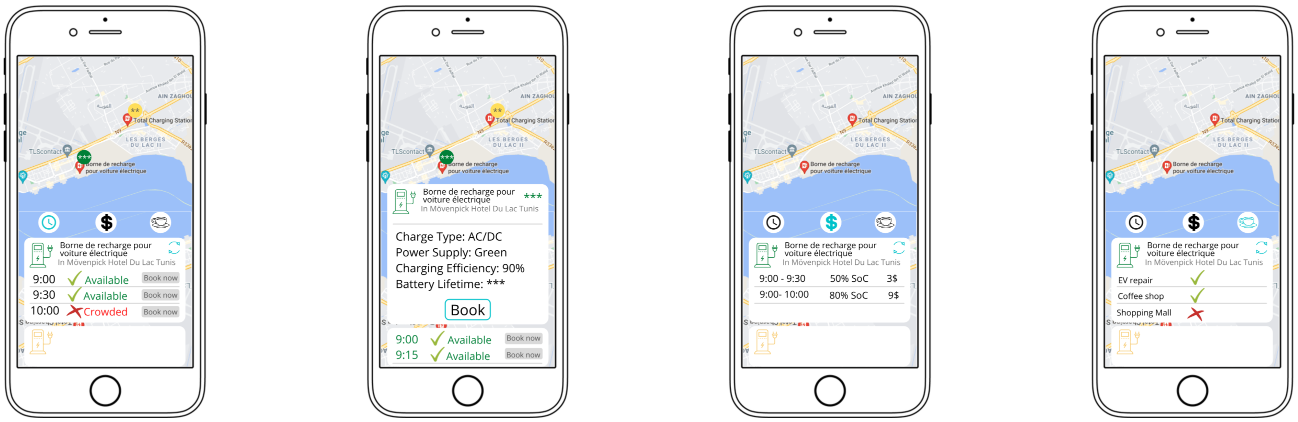


Fig. 1: Trip Planning App Reasoning.



(a) CS availability interface (b) CS info interface (c) Pricing interface (d) Facilities interface

Fig. 2: EV Trip-planning Mobile App Interfaces.

them with these criteria and more is beyond the scope of this paper. Yet, it is envisioned as a perspective for this work. Figure 2 illustrates the design of the proposed application via its main user interfaces.

V. Ontology Engineering Methodology

In this section, we summarize the main phases we went through when developing our ontology.

The first step was to identify the scope of our ontology and its purpose. Therefore, we conduct an analysis to understand the general context of E-Mobility and to identify the different stakeholders, as explained is Section III; namely by answering several basic questions, e.g., Who are we creating value for? and For what we are going to use the ontology?, etc. This analysis helped us define the terms and concepts associated with the e-mobility domain. Then, we determined the set of facts and specific terminology, e.g., the different attributes and properties that describe each concept, etc.

After gathering the necessary requirements, the second step is the implementation. During this phase, we identified the technologies we need for the development of the ontology such as OWL language. Further details can be found in Section VI. The output was validated thanks to reasoners, and the SPARQL queries were performed to answer the use cases and scenarios we defined in the requirements phase.

VI. Ontology Implementation and Evaluation

After we gathered the necessary requirements, we moved to the implementation phase. In this section, we explain the concepts and showcase examples of our ontology.

A. Ontology reuse

As mentioned above, we considered re-using existing relevant standards.

Through our literature review, mainly reported in Section II, we chose SAREF as a core ontology for our solution. First, it is an application-oriented reference ontology. Second, it encompasses several concepts related to the automotive domain namely through its extension SAREF4AUTO (s4auto), as already explained in Section 2. Other existing ontologies like Time, Geo ontology, and Opengis geosparql are also integrated to cover most of our requirements.

B. Ontology development

In this section, we use prefixes for each term and we refer to our own terms using the prefix Renewable energy-based Electro-MObility (REMO), as illustrated in the diagrams in Figures 3 and 4.

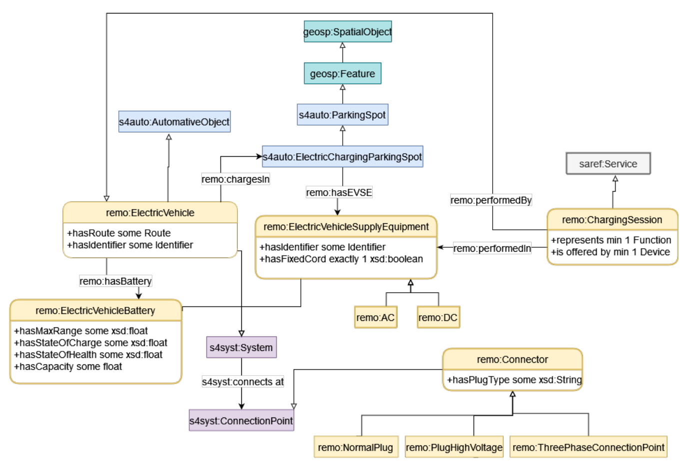


Fig. 3: Ontology main concepts.

The main concepts are modeled by extending the SAREF vocabulary, as described in Figure 3. Both the `remo:ElectricVehicle` and `remo:ElectricVehicleSupplyEquipment` have been introduced and are modeled as a `s4syst:System`, which `s4syst:connects at` the `s4syst :ConnectionPoint`. The `remo: ElectricVehicle` is also an `s4auto: AutomativeObject` that `remo:chargesIn` the `s4auto:ElectricChargingParkingSpot`. It has a `remo: ElectricVehicleBattery` with few properties (e.g., hasMaxRange, hasStateOfCharge and hasStateOfHealth). The concept

 ISBN: 978-1-61208-957-7

s4auto:ElectricChargingParkingSpot remo:hasEVSE remo: ElectricVehicleSupplyEquipment. The latter has two types: remo:AC and remo:DC. We also have remo:ChargingSession remo:performedBy the EV in the EVSE, and extends from saref:Service.

The EV and EVSE both connect at remo:Connector which extends from s4syst:ConnectionPoint. It also extends to three different types: remo: NormalPlug, remo:PlugHighVoltage, remo:Three PhaseConnectionPoint. It is important to distinguish between the different types since EVs have different connectors and not all EVSE offer all types. Each EVSE connects to at least a connection point.

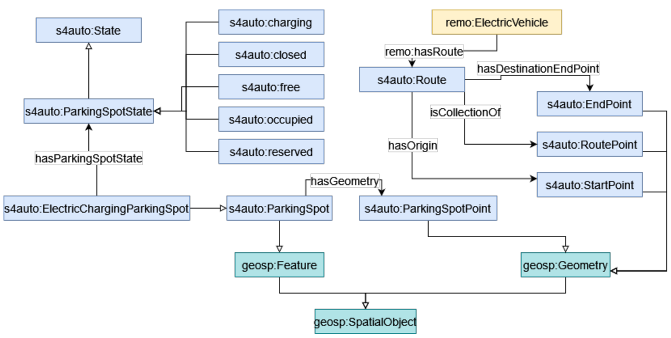


Fig. 4: Path planning concepts.

To address the path planning requirements, we need the s4auto:Route of the EV, as described in Figure 4. The remo:ElectricVehicle SupplyEquipment should be either on the s4auto:RoutePoint or at the s4auto:EndPoint. Also, we need the s4auto:State to verify if the CP is available for charging or not: e.g., s4auto:free and s4auto:occupied. Another important aspect is the time. For instance, each state saref:has timestamp at which it has started. For example, the CP can be free at 10:00 am and then occupied at 10:30 am.

C. Ontology usage examples

In this section, we showcase how our ontology answers the different criteria of our trip planning application using SPARQL query language.

1) Availability: We start with availability questions. As explained in Section IV, a CP is available when it is both on the EV route and has an ”available” state.

Is there a CP at the EV destination? For this, we need the EV route and its destination. Then, we check if there exists a CP point at the destination. In Figure 5a, we showcase the SPARQL query answering this first question.

Are there any CP on the EV route? To compose the query, we ask for the CPs that are on the route points of the EV route. In Figure 5b, we showcase the corresponding SPARQL query.

What are the CPs that can be reached from the EV start point within its maximum range? When the EV leaves its start point and is heading to the destination, it might need to recharge on the way. Therefore, it is important to know the CPs that can be reached before the EV Battery State Of Charge (SOC) drops below 10%, as explained in Section IV. In Figure 6a, we showcase the SPARQL query. Note that the EV maxRange property is measured by an on-board sensor available on electric vehicles.

When is the CP available? To answer this question, we need to know each state of the CP and its timeStamp and let the app user decide when to book a CP depending on the result. In Figure 6b, we report the corresponding SPARQL query.

2) Compatibility: This is the second criteria to be checked in our trip planning app. The EVSE of the CP has to be compatible with EV connector, as shown in Figure 7.

D. Ontology evaluation

To evaluate our ontology, we used OOPS! scanner [12] to detect pitfalls. Since we imported and reused existing vocabularies, the results show only one critical issue which is that our ontology is not deployed online. However, our project is still under development and this is not its final state, so we did not deploy it on the web yet. Additionally, to evaluate the consistency of the ontology, we used reasoners (Pellet reasoner in Protégé) and we found no issue. For further evaluation and maintenance, our ontology is available here [13].

VII. Conclusion and Future Directions

E-Mobility and eMaaS have been identified among the most promising disruptive technologies that would help migrate to carbon neutral transportation systems. Nevertheless, a lot of research issues are yet to be addressed to ensure their mass adoption. In this paper, we have proposed an ontology that covers the most relevant concepts related to E-Mobility and facilitates knowledge sharing among different actors. The work mainly addresses EV drivers and particularly the development of trip planning and charging points booking services to avoid the range anxiety. Further extensions are envisioned for a better coupling of EV drivers' preferences and for answering the needs of other stakeholders in the E-Mobility ecosystem. Moreover, we plan to integrate features that would ease the monitoring of the grid resources and optimize their use. While currently relying on datasets to populate the ontology, introducing APIs and data collected from sensors for the ontology population is the next step to consider.

Acknowledgement

This work was supported by the German Academic Exchange Service (DAAD), Federal Ministry for Economic Cooperation and Development (BMZ), Germany, within the framework of the REMO project (Renewable Energy-based E-Mobility in Higher Education) ID 575455.

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```
SELECT ?ev ?cp ?endpoint ?parkingpt
WHERE { ?ev remo:hasRoute ?route.
?ev remo:hasIdentifier s4auto:id123  .
?route s4auto:hasDestinationEndPoint ?endpoint.
?endpoint owl:sameAs ?parkingpt.
?cp geosp:hasGeometry ?parkingpt.
}
```

(a) SPARQL query to check CP in destination.

```
SELECT ?ev ?cp
WHERE { ?ev remo:hasRoute ?route.
?ev remo:hasIdentifier s4auto:id123  .
?route s4auto:isCollectionOf ?routept.
?cp geosp:hasGeometry ?parkingpt.
?parkingpt owl:sameAs ?routept.
}
```

(b) SPARQL query to check CP on the EV route.

Fig. 5: SPARQL queries to check CPs on the EV route and destination

```
SELECT ?cp ?range WHERE {
    ?ev remo:hasBattery ?battery.
    ?battery remo:hasMaxRange ?range.
    ?ev remo:hasRoute ?route.
    ?route s4auto:hasOrigin ?start.
    ?start geosp:asWKT ?wkt1.
    ?cp a s4auto:ElectricChargingParkingSpot.
    ?cp geosp:hasGeometry ?pt.
    ?pt geosp:asWKT ?wkt2.
    FILTER (geof:distance(?wkt1, ?wkt2, uom:metre) < ?range).
}
```

(a) SPARQL query to check EV maximum range.

```
SELECT ?cp ?state ?time
WHERE {
    ?ev s4auto:hasRoute ?route.
    ?ev s4auto:hasIdentifier s4auto:id123  .
    ?route s4auto:isCollectionOf ?routept.
    ?cp geosp:hasGeometry ?parkingpt.
    ?parkingpt owl:sameAs ?routept.
    ?cp s4auto:hasParkingSpotState ?state.
    ?state a s4auto:Free.
    ?state saref:hasTimestamp ?time.

}
```

(b) SPARQL query to check available CPs on the EV route.

Fig. 6: SPARQL queries to check reachable available CPs.

```
SELECT ?ev ?cp ?c WHERE {
    ?ev s4syst:connectsAt ?c.
    ?ev a remo:ElectricVehicle.
    ?evse s4syst:connectsAt ?c.
    ?cp remo:hasEVSE ?evse.
}
```

Fig. 7: SPARQL query to check compatibility between EV and EVSE.

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Zero-Sum Games with Distributionally Robust Chance Constraints

Nguyen Hoang Nam
L2S, Centrale Supelec
University Paris Saclay
Orsay, France
e-mail: hoang-nam.nguyen3@l2s.centralesupelec.fr

Vikas Vikram Singh
Department of Mathematics
Indian Institute of Technology Delhi
New Delhi, India
e-mail: vikassingh@iitd.ac.in

Abdel Lisser
L2S, Centrale Supelec
University Paris Saclay
Orsay, France
e-mail: abdel.lisser@l2s.centralesupelec.fr

Monika Arora
Department of Mathematics
Indraprastha Institute of Information Technology Delhi
New Delhi, India
e-mail: monika@iiitd.ac.in

***Abstract*—We consider a two-player zero-sum game with random linear chance constraints whose distributions are known to belong to moments based uncertainty sets. We show that a Saddle Point Equilibrium problem is equivalent to a primal-dual pair of Second-Order Cone Programs. The game with chance constraints can be used in various applications, e.g., risk constraints in portfolio optimization, resource constraints in stochastic shortest path problem, renewable energy aggregators in the local market.**

Keywords—Distributionally robust chance constraints, Zero-sum game, Saddle point equilibrium, Second-order cone program.

I. INTRODUCTION

A two-player zero-sum game is defined using a single payoff function, where one player plays the role of maximizer and another player plays the role of minimizer. More commonly, a zero-sum game is introduced with a payoff matrix, where the rows and the columns are the actions of player 1 and player 2, respectively. A Saddle Point Equilibrium (SPE) is the solution concept to study the zero-sum games and it exists in the mixed strategies [1]. Dantzig and later Adler showed the equivalence between linear programming problems and two-player zero-sum games [2] [3]. Charnes [4] generalized the zero-sum game considered in [1] by introducing linear inequality constraints on the mixed strategies of both the players and called it a constrained zero-sum game. An SPE of a constrained zero-sum game can be obtained from the optimal solutions of a primal-dual pair of linear programs [4]. Singh and Lisser [5] considered a stochastic version of constrained zero-sum game considered by Charnes [4], where the mixed strategies of each player are restricted by random linear inequality constraints, which are modelled using chance constraints. When the random constraint vectors follow a multivariate elliptically symmetric distribution, the zero-sum game problem is equivalent to a primal-dual pair of Second-Order Cone Programs (SOCPs) [5].

Nash equilibrium is the generalization of SPE and it is used as a solution concept for the general-sum games [6] [7]. Under certain conditions on payoff functions and strategy sets, there always exists a Nash equilibrium [8]. The general-sum games under uncertainties are considered in the literature [9]–[13], which capture both risk neutral and risk averse situations.

In this paper, we consider a more general two player zero-sum game as compared to [5]. Unlike in [5], the strategy set of each player is defined by a compact polyhedral set, which is further restricted by some random linear inequalities and the information on the distribution of the random constraint vectors is not exactly known. We consider two different uncertainty sets based on the partial information on the mean vectors and covariance matrices of the random constraint vectors. We show that, there exists an SPE of the game Z_α and an SPE problem is equivalent to a primal-dual pair of SOCPs.

The rest of the paper is organized as follows. The definition of a distributionally robust zero-sum game is given in Section II. Section III presents the reformulation of distributionally robust chance constraints as second order cone constraints under two different uncertainty sets. Section IV outlines a primal-dual pair of SOCPs whose optimal solutions constitute an SPE of the game.

II. THE MODEL

We consider a two player zero-sum game, where each player has continuous strategy set. Let $C^1 \in \mathbb{R}^{K_1 \times m}$, $C^2 \in \mathbb{R}^{K_2 \times n}$, $d^1 \in \mathbb{R}^{K_1}$ and $d^2 \in \mathbb{R}^{K_2}$. We consider $X = \{x \in \mathbb{R}^m \mid C^1 x = d^1, x \geq 0\}$ and $Y = \{y \in \mathbb{R}^n \mid C^2 y = d^2, y \geq 0\}$ as the strategy sets of player 1 and player 2, respectively. We assume that X and Y are compact sets. Let $u : X \times Y \to \mathbb{R}$ be a payoff function associated to the zero-sum game and we assume that player 1 (resp. player 2) is interested in maximizing (resp. minimizing) $u(x, y)$ for a fixed strategy y (resp. x) of player 2 (resp. player 1). For a given strategy pair $(x, y) \in X \times Y$, the payoff function $u(x, y)$ is given by

$$u(x, y) = x^T G y + g^T x + h^T y, \tag{1}$$

where $G \in \mathbb{R}^{m \times n}$, $g \in \mathbb{R}^m$ and $h \in \mathbb{R}^n$. The first term of (1) results from the interaction between both the players whereas the second and third term represents the individual impact of player 1 and player 2 on the game, respectively. The strategy sets are often restricted by random linear constraints, which are modelled using chance constraints. The chance constraint based strategy sets appear in many practical problems, e.g., risk constraints in portfolio optimization [14]. In this paper, we consider the case, where the strategies of player 1 satisfy the following random linear constraints,

$$(a_k^1)^T x \leq b_k^1,\ k = 1, 2, \dots, p, \tag{2}$$

whilst the strategies of player 2 satisfy the following random linear constraints

$$(a_l^2)^T y \geq b_l^2,\ l = 1, 2, \dots, q. \tag{3}$$

Let $\mathcal{I}_1 = \{1, 2, \dots, p\}$ and $\mathcal{I}_2 = \{1, 2, \dots, q\}$ be the index sets for the constraints of player 1 and player 2, respectively. For each $k \in \mathcal{I}_1$ and $l \in \mathcal{I}_2$, the vectors a_k^1 and a_l^2 are random vectors defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. We consider the case, where the only information we have about the distributions of a_k^1 and a_l^2 is that they belong to some uncertainty sets $\mathcal{D}_k^1$ and $\mathcal{D}_l^2$, respectively. The uncertainty sets $\mathcal{D}_k^1$ and $\mathcal{D}_l^2$, are constructed based on the partially available information on the distributions of a_k^1 and a_l^2, respectively. Using the worst case approach, the random linear constraints (2) and (3) can be formulated as distributionally robust chance constraints given by

$$\inf_{F_k^1 \in \mathcal{D}_k^1} \mathbb{P}\left((a_k^1)^T x \leq b_k^1\right) \geq \alpha_k^1,\ \forall\ k \in \mathcal{I}_1, \tag{4}$$

and

$$\inf_{F_l^2 \in \mathcal{D}_l^2} \mathbb{P}\left((-a_l^2)^T y \leq -b_l^2\right) \geq \alpha_l^2,\ \forall\ l \in \mathcal{I}_2, \tag{5}$$

where α_k^1 and α_l^2 are the confidence levels of player 1 and player 2 for kth and lth constraints, respectively. Therefore, for a given $\alpha^1 = (\alpha_k^1)_{k \in \mathcal{I}_1}$ and $\alpha^2 = (\alpha_l^2)_{l \in \mathcal{I}_2}$, the feasible strategy sets of player 1 and player 2 are given by

$$S_{\alpha^1}^1 = \left\{x \in X |\ \inf_{F_k^1 \in \mathcal{D}_k^1} \mathbb{P}\{(a_k^1)^T x \leq b_k^1\} \geq \alpha_k^1,\ \forall\ k \in \mathcal{I}_1\right\}, \tag{6}$$

and

$$S_{\alpha^2}^2 = \left\{y \in Y |\ \inf_{F_l^2 \in \mathcal{D}_l^2} \mathbb{P}\{(-a_l^2)^T y \leq -b_l^2\} \geq \alpha_l^2,\ \forall\, l \in \mathcal{I}_2\right\}. \tag{7}$$

We call the zero-sum game with the strategy set $S_{\alpha^1}^1$ for player 1 and the strategy set $S_{\alpha^2}^2$ for player 2 as a distributionally robust zero-sum game. We denote this game by Z_α. A strategy pair $(x^*, y^*) \in S_{\alpha^1}^1 \times S_{\alpha^2}^2$ is called an SPE of the game Z_α at $\alpha = (\alpha^1, \alpha^2) \in [0, 1]^p \times [0, 1]^q$, if

$$u(x, y^*) \leq u(x^*, y^*) \leq u(x^*, y),\ \forall\ x \in S_{\alpha^1}^1,\ y \in S_{\alpha^2}^2.$$

III. Reformulation of Distributionally Robust Chance Constraints

We consider two different uncertainty sets based on the partial information about the mean vectors and covariance matrices of the random constraint vectors a_k^i, $i = 1, 2$, $k \in \mathcal{I}_i$. For each uncertainty set, distributionally robust chance constraints (4) and (5) are reformulated as second-order cone constraints.

A. Moments Based Uncertainty Sets

For each player i, $i = 1, 2$, we consider the case, where the mean vector and covariance matrix of the random vector a_k^i for all $k \in \mathcal{I}_i$ are known to belong to polytopes $U_{\mu_k^i}$ and $U_{\Sigma_k^i}$, respectively. We consider polytopes $U_{\mu_k^i} = Conv(\mu_{k1}^i, \mu_{k2}^i, \dots, \mu_{kM}^i)$ and $U_{\Sigma_k^i} = Conv(\Sigma_{k1}^i, \Sigma_{k2}^i, \dots, \Sigma_{kM}^i)$, where $\Sigma_{kj}^i \succ 0$, $j = 1, 2, \dots M$; $Conv$ denotes the convex hull and $\Sigma_{kj}^i \succ 0$ implies that Σ_{kj}^i is a positive definite matrix. For each $i = 1, 2$, and $k \in \mathcal{I}_i$, the uncertainty set for the distribution of a_k^i is defined by

$$\mathcal{D}_k^i\left(\mu_k^i, \Sigma_k^i\right) = \left\{F_k^i \ \middle|\ \begin{array}{l} E_{F_k^i}\left[a_k^i\right] \in U_{\mu_k^i} \\ COV_{F_k^i}[a_k^i] \in U_{\Sigma_k^i} \end{array} \right\}, \tag{8}$$

where $E_{F_k^i}$ and $COV_{F_k^i}$ are expectation and covariance operator under probability distribution F_k^i, respectively. The uncertainty set (8) is considered in [15]. As for the second uncertainty set, we consider the case, where the mean vector of a_k^i lies in an ellipsoid of size $\gamma_{k1}^i \geq 0$ centered at μ_k^i and the covariance matrix of a_k^i lies in a positive semidefinite cone defined with a linear matrix inequality. It is defined by

$$\mathcal{D}_k^i(\mu_k^i, \Sigma_k^i) = \left\{F_k^i \ \middle|\ \begin{array}{l} \left(\mathbb{E}_{F_k^i}[a_k^i] - \mu_k^i\right)^\top \left(\Sigma_k^i\right)^{-1} \\ \times \left(\mathbb{E}_{F_k^i}[a_k^i] - \mu_k^i\right) \leq \gamma_{k1}^i, \\ \\ COV_{F_k^i}[a_k^i] \preceq \gamma_{k2}^i \Sigma_k^i \end{array} \right\}, \tag{9}$$

where $\Sigma_k^i \succ 0$ and $\gamma_{k2}^i > 0$; for the given matrices B_1 and B_2, $B_1 \preceq B_2$ implies that $B_2 - B_1$ is a positive semidefinite matrix. The uncertainty set (9) is considered in [16].

B. Second-order cone constraint reformulation

We show that the distributionally robust chance constraints (4) and (5) are equivalent to second-order cone constraints for the uncertainty sets defined by (8) and (9).

Lemma III.1. *For each $i = 1, 2$, and $k \in \mathcal{I}_i$, let the distribution F_k^i of random vector a_k^i, lies in uncertainty set $\mathcal{D}_k^i\left(\mu_k^i, \Sigma_k^i\right)$ defined by* (8). *Then, the constraints* (4) *and* (5) *are equivalent to* (10) *and* (11), *respectively, given by*

$$\begin{aligned} &(\mu_{kj}^1)^T x + \sqrt{\frac{\alpha_k^1}{1 - \alpha_k^1}} ||(\Sigma_{kj}^1)^{\frac{1}{2}} x|| \leq b_k^1, \\ &\forall\ j = 1, 2, \dots, M,\ k \in \mathcal{I}_1, \end{aligned} \tag{10}$$

$$\begin{aligned} &-(\mu_{kj}^2)^T y + \sqrt{\frac{\alpha_k^2}{1 - \alpha_k^2}} ||(\Sigma_{kj}^2)^{\frac{1}{2}} y|| \leq -b_k^2, \\ &\forall\ j = 1, 2, \dots, M,\ k \in \mathcal{I}_2. \end{aligned} \tag{11}$$

Proof. Based on the structure of uncertainty set (8), (4) can be written as

$$\inf_{(\mu, \Sigma) \in \mathcal{U}_k^1} \inf_{F_k^1 \in \mathcal{D}(\mu, \Sigma)} \mathbb{P}\left\{(a_k^1)^T x \leq b_k^1\right\} \geq \alpha_k^1,$$

where

$$\mathcal{D}(\mu,\Sigma)=\left\{F_k^1 \,\middle|\, \mathbb{E}_{F_k^1}[a_k^1]=\mu, COV_{F_k^1}[a_k^1]=\Sigma\right\},$$

and

$$\mathcal{U}_k^1=\left\{(\mu,\Sigma) \,\middle|\, \mu\in U_{\mu_k^1}, \Sigma\in U_{\Sigma_k^1}\right\}.$$

The bound of one-sided Chebyshev inequality can be achieved by a two-point distribution given by equation (2) of [17]. Therefore, we have

$$\inf_{F_k^1\in\mathcal{D}(\mu,\Sigma)} \mathbb{P}\{(a_k^1)^T x\le b_k^1\}=\begin{cases}1-\frac{1}{1+\frac{(\mu^T x-b_k^1)^2}{(x^T\Sigma x)}}, \\ \text{if } \ \mu^T x\le b_k^1, \\ \\ 0, \ \text{otherwise.}\end{cases}$$

For the case $\mu^T x > b_k^1$,

$$\inf_{F_k^1\in\mathcal{D}(\mu,\Sigma)} \mathbb{P}\{a_k^1 x\le b_k^1\}=0,$$

which makes constraint (4) infeasible for any $\alpha_1 > 0$. Therefore, for $x\in S_{\alpha_1}^1$ the condition $\mu^T x\le b_k^1$ always holds and the constraint (4) is equivalent to

$$\inf_{(\mu,\Sigma)\in\mathcal{U}_k^1} 1-\frac{1}{1+(\mu^T x-b_k^1)^2/(x^T\Sigma x)}\ge\alpha_k^1,$$

which can be reformulated as

$$\frac{\min\limits_{\mu\in U_{\mu_k^1}} \left(b_k^1-\mu^T x\right)}{\max\limits_{\Sigma\in U_{\Sigma_k^1}} \sqrt{x^T\Sigma x}}\ge\sqrt{\frac{\alpha_k^1}{1-\alpha_k^1}}. \tag{12}$$

The above inequality (12) can be reformulated as (10). Similarly, we can show that (5) is equivalent to (11). □ □

Lemma III.2. *For each $i=1,2$, and $k\in\mathcal{I}_i$, let the distribution F_k^i of random vector a_k^i, lies in the uncertainty set $\mathcal{D}_k^i\left(\mu_k^i,\Sigma_k^i\right)$ defined by* (9)*. Then, the constraints* (4) *and* (5) *are equivalent to* (13) *and* (14)*, respectively, given by*

$$(\mu_k^1)^T x+\left(\sqrt{\frac{\alpha_k^1}{1-\alpha_k^1}}\sqrt{\gamma_{k2}^1}+\sqrt{\gamma_{k1}^1}\right)\left\|\left(\Sigma_k^1\right)^{\frac{1}{2}}x\right\|\le b_k^1,$$
$$\forall\ k\in\mathcal{I}_1, \tag{13}$$

$$-(\mu_k^2)^T y+\left(\sqrt{\frac{\alpha_k^2}{1-\alpha_k^2}}\sqrt{\gamma_{k2}^2}+\sqrt{\gamma_{k1}^2}\right)\left\|\left(\Sigma_k^2\right)^{\frac{1}{2}}y\right\|\le -b_k^2,$$
$$\forall\ k\in\mathcal{I}_2. \tag{14}$$

Proof. Based on the structure of the uncertainty set (9), the constraint (4) can be written as

$$\inf_{(\mu,\Sigma)\in\tilde{\mathcal{U}}_k^1}\ \inf_{F_k^1\in\mathcal{D}(\mu,\Sigma)} \mathbb{P}\{a_k^1 x\le b_k^1\}\ge\alpha_k^1,$$

where

$$\mathcal{D}(\mu,\Sigma)=\left\{F_k^1 \,\middle|\, \mathbb{E}_{F_k^1}[a_k^1]=\mu, COV_{F_k^1}[a_k^1]=\Sigma\right\}$$

and

$$\tilde{\mathcal{U}}_k^1=\left\{(\mu,\Sigma) \,\middle|\, \begin{array}{l}\left(\mu-\mu_k^1\right)^\top\left(\Sigma_k^1\right)^{-1}\left(\mu-\mu_k^1\right)\le\gamma_{k1}^1, \\ \Sigma\preceq\gamma_{k2}^1\Sigma_k^1.\end{array}\right\}.$$

Using the similar arguments as in the Lemma III.1, the constraint (4) is equivalent to

$$\frac{b_k^1+v_1(x)}{\sqrt{v_2(x)}}\ge\sqrt{\frac{\alpha_k^1}{1-\alpha_k^1}}, \tag{15}$$

where

$$v_1(x)=\begin{cases}\min\limits_{\mu} -\mu^T x \\ \text{s.t.}\ \ \left(\mu-\mu_k^1\right)^\top\left(\Sigma_k^1\right)^{-1}\left(\mu-\mu_k^1\right)\le\gamma_{k1}^1,\end{cases} \tag{16}$$
$$v_2(x)=\begin{cases}\max\limits_{\Sigma}\ x^T\Sigma x \\ \text{s.t. } \Sigma\preceq\gamma_{k2}^1\Sigma_k^1.\end{cases}$$

Let $\beta\ge 0$ be a Lagrange multiplier associated with the constraint of optimization problem (16). By applying the KKT conditions, the optimal solution of (16) is given by $\mu=\mu_k^1+\frac{\sqrt{\gamma_{k1}^1}\Sigma_k^1 x}{\sqrt{x^T\Sigma_k^1 x}}$ and the associated Lagrange multiplier is given by $\beta=\sqrt{\frac{x^T\Sigma_k^1 x}{4\gamma_{k1}^1}}$. Therefore, the corresponding optimal value $v_1(x)=-(\mu_k^1)^T x-\sqrt{\gamma_{k1}^1}\sqrt{x^T\Sigma_k^1 x}$. Since, $u^T\Sigma u\le u^T\gamma_{k2}^1\Sigma_k^1 u$ for any $u\in\mathbb{R}^n$, then, $v_2(x)=\gamma_{k2}^1 x^T\Sigma_k^1 x$. Therefore, using (15), (4) is equivalent to (13). Similarly, we can show that (5) is equivalent to (14). □ □

The reformulation of feasible strategy sets (6) and (7) for uncertainty sets (8) and (9) can be written as

$$S_{\alpha^1}^1=\Big\{x\in X \mid (\mu_{kj}^1)^T x+\kappa_{\alpha_k^1}||(\Sigma_{kj}^1)^{\frac{1}{2}}x||\le b_k^1,$$
$$\forall\ j=1,2,\ldots,M,\ k\in\mathcal{I}_1\Big\}, \tag{17}$$

and

$$S_{\alpha^2}^2=\Big\{y\in Y \mid -(\mu_{lj}^2)^T y+\kappa_{\alpha_l^2}||(\Sigma_{lj}^2)^{\frac{1}{2}}y||\le -b_l^2,$$
$$\forall\ j=1,2,\ldots,M,\ l\in\mathcal{I}_2.\Big\}. \tag{18}$$

For each $i=1,2$, $k\in\mathcal{I}_i$, if $\kappa_{\alpha_k^i}=\sqrt{\frac{\alpha_k^i}{1-\alpha_k^i}}$, (17) and (18) represent the reformulations of (6) and (7) under uncertainty set defined by (8), respectively. For each $i=1,2$, $k\in\mathcal{I}_i$, if $\kappa_{\alpha_k^i}=\left(\sqrt{\frac{\alpha_k^i}{1-\alpha_k^i}}\sqrt{\gamma_{k2}^i}+\sqrt{\gamma_{k1}^i}\right)$, and $M=1$, (17) and (18) represent the reformulations of (6) and (7) under uncertainty set defined by (9), respectively.

We assume that the strategy sets (17) and (18) satisfy the strict feasibility condition given by Assumption III.3.

Assumption III.3.
1) *There exists an $x\in S_{\alpha^1}^1$ such that the inequality constraints of $S_{\alpha^1}^1$ defined by* (17) *are strictly satisfied.*
2) *There exists an $y\in S_{\alpha^2}^2$ such that the inequality constraints of $S_{\alpha^2}^2$ defined by* (18) *are strictly satisfied.*

The conditions given in Assumption III.3 are Slater's condition, which are sufficient for strong duality in a convex optimization problem. We use these conditions in order to derive equivalent SOCPs for the zero-sum game Z_α.

IV. Existence and characterization of Saddle Point Equilibrium

In this section, we show that there exists an SPE of the game Z_α if the distributions of the random constraint vectors of both the players belong to the uncertainty sets defined in Section III-A. We further propose a primal-dual pair of SOCPs whose optimal solutions constitute an SPE of the game Z_α.

Theorem IV.1. *Consider the game Z_α, where the distributions of the random constraint vectors a_k^i, $k \in \mathcal{I}_i$, $i = 1, 2$, belong to the uncertainty sets described in Section III-A. Then, there exists an SPE of the game for all $\alpha \in (0,1)^p \times (0,1)^q$.*

Proof. Let $\alpha \in (0,1)^p \times (0,1)^q$. For uncertainty sets described in Section III-A, it follows from Lemma III.1 and Lemma III.2 that the strategy sets $S^1_{\alpha^1}$ and $S^2_{\alpha^2}$ are given by (17) and (18), respectively. It is easy to see that $S^1_{\alpha^1}$ and $S^2_{\alpha^2}$ are convex and compact sets. The function $u(x,y)$ is a bilinear and continuous function. Hence, there exists an SPE from the minimax theorem [1]. □

A. Equivalent Primal-Dual Pair of Second-Order Cone Programs

From the minimax theorem [1], (x^*, y^*) is an SPE for the game Z_α if and only if

$$x^* \in \arg\max_{x \in S^1_{\alpha^1}} \min_{y \in S^2_{\alpha^2}} u(x,y), \tag{19}$$

$$y^* \in \arg\min_{y \in S^2_{\alpha^2}} \max_{x \in S^1_{\alpha^1}} u(x,y). \tag{20}$$

We start with problem $\min_{y\in S^2_{\alpha^2}} \max_{x \in S^1_{\alpha^1}} u(x,y)$. The inner optimization problem $\max_{x\in S^1_{\alpha^1}} u(x,y)$ can be equivalently written as

$$\begin{aligned}
&\max_{x, t^1_{k,j}} x^T G y + g^T x + h^T y \\
&\text{s.t.} \\
&(i)\ -x^T \mu^1_{k,j} - \kappa_{\alpha^1_k} \left\| t^1_{k,j} \right\| + b^1_k \geq 0, \\
&\forall\ j = 1, 2 \ldots, M,\ k \in \mathcal{I}_1, \\
&(ii)\ t^1_{k,j} - \left(\Sigma^1_{k,j}\right)^{\frac{1}{2}} x = 0,\ \forall\ j = 1, 2 \ldots, M,\ k \in \mathcal{I}_1, \\
&(iii)\ C^1 x = d^1,\ x_r \geq 0,\ \forall\ r = 1, 2, \ldots, m.
\end{aligned} \tag{21}$$

Let $\lambda^1 = \left(\lambda^1_{k,j}\right)^M_{j=1, k\in\mathcal{I}_1} \in \mathbb{R}^{Mp}$, $\delta^1_{k,j} \in \mathbb{R}^m$ for all $k \in \mathcal{I}_1$, $j = 1, 2 \ldots M$, and $\nu^1 \in \mathbb{R}^{K_1}$ be the Lagrange multipliers of constraints $(i), (ii)$, and equality constraints given in (iii) of (21), respectively. Then, the Lagrangian dual problem of the SOCP (21) can be written as

$$\begin{aligned}
&\min_{\lambda_1 \geq 0,\ \delta^1_{k,j},\ \nu^1} \ \max_{x \geq 0,\ t^1_{k,j}} \Big\{ x^T G y + g^T x + h^T y \\
&+ \sum_{k\in\mathcal{I}_1} \sum_{j=1}^M \Big[\lambda^1_{k,j} \big(-x^T \mu^1_{k,j} - \kappa_{\alpha^1_k} \left\| t^1_{k,j} \right\| + b^1_k \big) \\
&+ (\delta^1_{k,j})^T \Big(t^1_{k,j} - \left(\Sigma^1_{k,j}\right)^{\frac{1}{2}} x \Big) \Big] + (\nu^1)^T (d^1 - C^1 x) \Big\} \\
&= \min_{\lambda_1 \geq 0, \delta^1_{k,j}, \nu^1} \Big[\max_{x \geq 0}\ x^T \Big(G y \\
&- \sum_{k\in\mathcal{I}_1} \sum_{j=1}^M \left(\lambda^1_{k,j} \mu^1_{k,j} + \left(\Sigma^1_{k,j}\right)^{\frac{1}{2}} \delta^1_{k,j} \right) - (C^1)^T \nu^1 + g \Big) \\
&+ \sum_{k\in\mathcal{I}_1} \sum_{j=1}^M \max_{t^1_{k,j}} \ \Big((\delta^1_{k,j})^T t^1_{k,j} - \kappa_{\alpha^1_k} \lambda^1_{k,j} \left\| t^1_{k,j} \right\| \Big) + h^T y \\
&+ (\nu^1)^T d^1 + \sum_{k\in\mathcal{I}_1} \sum_{j=1}^M \lambda^1_{k,j} b^1_k \Big].
\end{aligned}$$

The inner maximization problems in the above Lagrangian dual problem will be unbounded unless we have the following dual constraints

$$\begin{aligned}
&G y - \sum_{k\in\mathcal{I}_1} \sum_{j=1}^M \left(\lambda^1_{k,j} \mu^1_{k,j} + \left(\Sigma^1_{k,j}\right)^{\frac{1}{2}} \delta^1_{k,j} \right) \\
&- (C^1)^T \nu^1 + g \leq 0, \\
&||\delta^1_{k,j}|| \leq \kappa_{\alpha^1_k} \lambda^1_{k,j}, \forall\ k \in \mathcal{I}_1, j = 1, 2 \ldots, M.
\end{aligned}$$

Under Assumption III.3, the Lagrangian dual problem of (21) has zero duality gap [18]. Therefore, the problem $\min_{y\in S^2_{\alpha^2}} \max_{x\in S^1_{\alpha^1}} u(x,y)$ is equivalent to the following SOCP

$$\begin{aligned}
&\min_{y,\ \nu^1,\ \delta^1_{k,j},\ \lambda^1_{k,j}} \quad h^T y + (\nu^1)^T d^1 + \sum_{k\in\mathcal{I}_1} \sum_{j=1}^M \lambda^1_{k,j} b^1_k \\
&\text{s.t.} \\
&(i)\ G y - \sum_{k\in\mathcal{I}_1} \sum_{j=1}^M \left(\lambda^1_{k,j} \mu^1_{k,j} + \left(\Sigma^1_{k,j}\right)^{\frac{1}{2}} \delta^1_{k,j} \right) \\
&- (C^1)^T \nu^1 + g \leq 0, \\
&(ii)\ -(\mu^2_{lj})^T y + \kappa_{\alpha^2_l} ||(\Sigma^2_{lj})^{\frac{1}{2}} y|| \leq -b^2_l, \\
&\forall\ j = 1, 2, \ldots, M,\ l \in \mathcal{I}_2, \\
&(iii)\ ||\delta^1_{k,j}|| \leq \kappa_{\alpha^1_k} \lambda^1_{k,j},\ \lambda^1_{k,j} \geq 0, \\
&\forall\ k \in \mathcal{I}_1,\ j = 1, 2 \ldots, M, \\
&(iv)\ C^2 y = d^2,\ y_s \geq 0,\ \forall\ s = 1, 2, \ldots, n.
\end{aligned} \tag{22}$$

Similarly, problem $\max_{x\in S^1_{\alpha^1}} \min_{y\in S^2_{\alpha^2}} u(x,y)$ is equivalent to the following SOCP

$$\max_{x,\ \nu^2,\ \delta^2_{l,j},\ \lambda^2_{l,j}} \ g^T x + (\nu^2)^T d^2 + \sum_{l\in\mathcal{I}_2}\sum_{j=1}^{M} \lambda^2_{l,j} b^2_l$$

$$\text{s.t. } (i)\ G^T x - \sum_{l\in\mathcal{I}_2}\sum_{j=1}^{M}\left(\lambda^2_{l,j}\mu^2_{l,j} + \left(\Sigma^2_{l,j}\right)^{\frac{1}{2}}\delta^2_{l,j}\right)$$
$$-(C^2)^T\nu^2 + h \geq 0,$$
$$(ii)\ (\mu^1_{kj})^T x + \kappa_{\alpha^1_k}||(\Sigma^1_{kj})^{\frac{1}{2}}x|| \leq b^1_k,$$
$$\forall\ j=1,2,\ldots,M,\ k\in\mathcal{I}_1,$$
$$(iii)\ ||\delta^2_{l,j}|| \leq \kappa_{\alpha^2_l}\lambda^2_{l,j},\ \lambda^2_{l,j}\geq 0,\ \forall\ \ l\in\mathcal{I}_2,\ j=1,2,\ldots M$$
$$(iv)\ C^1 x = d^1,\ x_r\geq 0,\ \forall\ r=1,2,\ldots,m. \quad (23)$$

It follows from the duality theory of SOCPs that (22) and (23) form a primal-dual pair [18].

Remark IV.2. *For each $i=1,2,$ and $k\in\mathcal{I}_i$, if $\kappa_{\alpha^i_k} = \sqrt{\frac{\alpha^i_k}{1-\alpha^i_k}}$,* (22) *and* (23) *represent the primal-dual pair of SOCPs for the uncertainty sets defined by* (8)*. For each $i=1,2,$ and $k\in\mathcal{I}_i$, if $\kappa_{\alpha^i_k} = \left(\sqrt{\frac{\alpha^i_k}{1-\alpha^i_k}}\sqrt{\gamma^i_{k2}}+\sqrt{\gamma^i_{k1}}\right)$ and $M=1$,* (22) *and* (23) *represent the primal-dual pair of SOCPs for the uncertainty set defined by* (9)*.*

Next, we show that the equivalence between the optimal solutions of (22)-(23) and an SPE of the game Z_α.

Theorem IV.3. *Consider the zero-sum game Z_α, where the feasible strategy sets of player 1 and player 2 are given by* (17) *and* (18)*, respectively. Let Assumption III.3 holds. Then, for a given $\alpha\in(0,1)^p\times(0,1)^q$, (x^*,y^*) is an SPE of the game Z_α if and only if there exists $(\nu^{1*},(\delta^{1*}_{k,j})_{k,j},\lambda^{1*})$ and $(\nu^{2*},(\delta^{2*}_{l,j})_{l,j},\lambda^{2*})$ such that $(y^*,\nu^{1*},(\delta^{1*}_{k,j})_{k,j},\lambda^{1*})$ and $(x^*,\nu^{2*},(\delta^{2*}_{l,j})_{l,j},\lambda^{2*})$ are optimal solutions of* (22) *and* (23)*, respectively.*

Proof. Let (x^*,y^*) be an SPE of the game Z_α. Then, x^* and y^* are the solutions of (19) and (20), respectively. Therefore, there exists $(\nu^{1*},(\delta^{1*}_{k,j})_{k,j},\lambda^{1*})$ and $(\nu^{2*},(\delta^{2*}_{l,j})_{l,j},\lambda^{2*})$ such that $(y^*,\nu^{1*},(\delta^{1*}_{k,j})_{k,j},\lambda^{1*})$ and $(x^*,\nu^{2*},(\delta^{2*}_{l,j})_{l,j},\lambda^{2*})$ are optimal solutions of (22) and (23) respectively.

Let $(y^*,\nu^{1*},(\delta^{1*}_{k,j})_{k,j},\lambda^{1*})$ and $(x^*,\nu^{2*},(\delta^{2*}_{l,j})_{l,j},\lambda^{2*})$ be optimal solutions of (22) and (23), respectively. Under Assumption III.3, (22) and (23) are strictly feasible. Therefore, strong duality holds for primal-dual pair (22)-(23). Then, we have

$$g^T x^* + (\nu^{2*})^T d^2 + \sum_{l\in\mathcal{I}_2}\sum_{j=1}^{M}\lambda^{2*}_{l,j}b^2_l = h^T y^*$$
$$+(\nu^{1*})^T d^1 + \sum_{k\in\mathcal{I}_1}\sum_{j=1}^{M}\lambda^{1*}_{k,j}b^1_k. \quad (24)$$

Consider the constraint (i) of (22) at optimal solution $(y^*,\nu^{1*},(\delta^{1*}_{k,j})_{k,j},\lambda^{1*})$ and multiply it by x^T, where $x\in S^1_{\alpha_1}$. Then, by using Cauchy-Schwartz inequality, we have

$$x^T G y^* + g^T x + h^T y^* \leq h^T y^* + (\nu^{1*})^T d^1$$
$$+\sum_{k\in\mathcal{I}_1}\sum_{j=1}^{M}\lambda^{1*}_{k,j}b^1_k,\ \forall\ x\in S^1_{\alpha_1}. \quad (25)$$

Similarly, we have

$$x^{*T} G y + g^T x^* + h^T y \geq g^T x^*$$
$$+(\nu^{2*})^T d^2 + \sum_{l\in\mathcal{I}_2}\sum_{j=1}^{M}\lambda^{2*}_{l,j}b^2_l,\ \forall\ y\in S^2_{\alpha_2}. \quad (26)$$

Take $x=x^*$ and $y=y^*$ in (25) and (26), then from (24), we get

$$u(x^*,y^*) = h^T y^* + (\nu^{1*})^T d^1 + \sum_{k\in\mathcal{I}_1}\sum_{j=1}^{M}\lambda^{1*}_{k,j}b^1_k$$
$$= g^T x^* + (\nu^{2*})^T d^2 + \sum_{l\in\mathcal{I}_2}\sum_{j=1}^{M}\lambda^{2*}_{l,j}b^2_l. \quad (27)$$

It follows from (25), (26), and (27) that (x^*,y^*) is an SPE of the game Z_α. □ □

V. Conclusion

We show the existence of a mixed strategy SPE for a two-player distributionally robust zero-sum chance-constrained game under three different uncertainty sets based on first two moments. Under Slater's condition, the Saddle Point Equilibria of the game can be obtained from the optimal solutions of a primal-dual pair of SOCPs. The Saddle Point Equilibria of zero-sum games can be computed efficiently because SOCPs are polynomial time solvable. The uncertainty sets considered in the paper have positive semidefinite cone structure, which leads to the reformulation of distributionally robust chance constraints as second order cone constraints. Moreover, these reformulations play a major role in deriving the equivalent primal-dual pair of SOCPs. The tractable reformulation of the zero-sum game problem with different payoff structure, as well as the uncertainty sets other than the ones considered in the paper could be an interesting area for the future research.

Acknowledgement

This research was supported by DST/CEFIPRA Project No. IFC/4117/DST-CNRS-5th call/2017-18/2 and CNRS Project No. AR/SB:2018-07-440.

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 ISBN: 978-1-61208-957-7